

NON-DIMENSIONAL RESPONSE SURFACES
FOR STRUCTURAL OPTIMIZATION WITH UNCERTAINTY

By

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This dissertation is dedicated to Sanet.

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Approximation concepts are an effective approach for alleviating some of the problems associated with the direct use of modern computerized analysis techniques in an optimization environment. Recently, response surface approximations have gained popularity as polynomial approximations that are global in nature. Response surface approximations shift the computational burden from the optimization problem to the problem of constructing the approximations, and accommodate the use of detailed analysis techniques without the need of derivative information. Additionally, response surface approximations filter out numerical noise inherent to most numerical analysis procedures, by providing a smooth approximate response function, and simplify the integration of the analysis and the optimization codes.

The present dissertation investigates the use of response surface approximations in expensive structural optimization problems and aims to suggest techniques for improving

both the accuracy of response surface approximations as well as the efficiency with which they are constructed. A stepped plate design problem is considered and response surface approximations are constructed for different failure mechanisms using numerical experiments conducted with a finite element analysis. Both an isotropic and a composite laminated plate, where the change in thickness is a result of internal ply drop off, are considered.

The proposed methodology uses a combination of dimensional analysis, higher order response surface approximations, stepwise regression, a detailed error analysis and statistical design of experiments to improve both accuracy and efficiency. Dimensional analysis identifies variables intrinsic to the problem, and thus reduces the number of variables in the resulting response surface approximation. Stepwise regression is used to eliminate insignificant parameters from a response surface approximation and statistical design of experiments is used to identify a small set of data points for constructing the response surface approximations, while maintaining accuracy. The result of the proposed methodology is response surface approximations that are highly accurate over the entire design space. The use of response surface approximations in expensive structural optimization problems is demonstrated by using the developed response surface approximations in designing for uncertainty. The uncertainty is modeled using fuzzy set theory and the optimization results are presented in the form of design charts.

CHAPTER 1 INTRODUCTION

1.1. Aspects of Modern Engineering Optimization

Modern computerized analysis techniques, in the form of numerical simulations, enable the detailed analysis of complex systems. Although detailed numerical simulations provide an invaluable tool in analyzing complex systems, several problem areas are associated with the direct use thereof in an optimization environment. Most notable is that the designer is provided with only a single design point and lacks an overall perspective of the system behavior within the design space. Without this overall perspective, complications may arise during the optimization procedure.

Inherent in most numerical analyses are human errors resulting from dealing with large, complex and cumbersome codes (Kaufman et al.¹ 1996), and numerical noise. Examples of numerical noise are automatic mesh generation procedures that provide finite element meshes with an inconsistent level of fidelity and round-off error (Burden and Faires² 1989, p. 10). Since human and numerical errors of substantial magnitude are usually detected and corrected, the remaining inconsistencies have negligible influence on the accuracy of the results, but may cause non-smooth functional behavior creating spurious local optima, thus making it difficult to find the best design (e.g., Giunta et al.³ 1994 and Dudley et al.⁴ 1995).

Another problem area arises when a designer needs to integrate several technical disciplines, as is typically required in multidisciplinary design optimization. Integrating several software codes poses a significant organizational challenge and is a complex and often

frustrating task (Kaufman et al.¹ 1996). Even though the problem of integrating different software codes is amplified in multidisciplinary optimization applications, it may also be an important factor in single discipline optimization, where it is difficult to link the optimization algorithm with the analysis code.

Finally, some optimization problems such as global optimization, multi-objective optimization and optimization under uncertainty require a very large number of analyses and are, except for simple problems, often impractical. Advances in parallel computing, which is an important aspect of modern computer systems, may help alleviate this problem. However, except for some special cases where multiple analyses are done simultaneously with very little communication between the processors (coarse grain parallelism), time consuming changes in the analysis code are often required in order to take advantage of the parallelism.

Detailed numerical simulations thus exhibit many of the same characteristics of physical experiments. A physical experiment provides the designer with only a single design point in the design space and is prone to experimental error. Additionally, it is difficult to integrate physical experiments with an optimization algorithm and it is typically most cost efficient to conduct physical experiments in large batches, consisting of a number of independent experiments (coarse grain parallelism). These similarities between detailed numerical simulations and physical experiments are discussed in more detail in a recent survey paper by Haftka, Scott and Cruz⁵ (1998) that concentrates on optimization in relationship to experiments.

1.2. Response Surface Methodology

Response surface methodology may be summarized as a collection of statistical tools and techniques for constructing and exploring an approximate functional relationship between

a response variable and a set of design variables. This approximate functional relationship is typically constructed in the form of a low order polynomial, referred to as a response surface approximation. Response surface methodology was originally developed for constructing and exploring approximate response functions based on physical experiments and results in smooth approximate response functions, thus effectively filtering out any noise. Response surface approximations allow easy integration between the experiments and the optimization algorithm, allow the evaluation of experiments in batches and provide the designer with an overall perspective of the system behavior within the design space. Due to the similarities between detailed numerical simulations and physical experiments, there has recently been growing interest in using response surface approximations within structural optimization.

Response surface methodology includes methods for selecting data points where the experiments should be evaluated (a process known as statistical design of experiments), methods for solving the unknown coefficients of a response surface approximation and methods for evaluating the accuracy of the resulting response surface approximation. The unknown coefficients of a response surface approximation are estimated from experimental data points by means of a process known as linear regression. These coefficients are estimated in such a way as to minimize the sum of the squares of the error terms (e.g., Myers and Montgomery⁶ 1995, pp. 16-27 and Box and Draper⁷ 1987, pp. 35-70). The accuracy of a response surface approximation is expressed in terms of various error terms and statistical parameters that are representative of the predictive capabilities of the approximation (e.g., Myers and Montgomery⁶ 1995, pp. 27-54).

Statistical design of experiments is used to select a small set of data points for constructing the response surface approximation without compromising accuracy.

Traditionally, statistical design of experiments is performed using a minimum-variance criterion that assumes all error to be random error or variance. For regularly shaped design spaces (i.e., rectangular or spherical regions) there exist a number of standard designs (i.e., sets of data points), for example the Central Composite (Box and Wilson⁸ 1951) and the Box-Behnken (Box and Behnken⁹ 1960) designs. For an irregularly shaped design space, a computer-generated design must be used, of which the most popular employ the *D*-Optimality design criterion (e.g., Myers and Montgomery⁶ 1995, pp. 364-366).

Alternatively, the data points may be selected to minimize the bias error of the resulting response surface approximations. These designs are known as minimum-bias designs. Bias error occurs when a response surface approximation that is inadequate in modeling the actual response function is used. Since the exact functional form of the response function to be approximated is rarely known and low order polynomials are generally used as response surface approximations, large bias error is easily introduced. For regularly shaped design spaces, analytical solutions exist for finding minimum-bias designs (Myers and Montgomery⁶ 1995, pp. 406-421 and Box and Draper⁷ 1987, pp. 437-442). Balabanov et al.¹⁰ (1996) successfully applied these analytical solutions to find a minimum-bias design for a 25-dimensional sphere. For irregularly shaped design spaces, no closed form solution exists and the minimum-bias data points must be obtained numerically (e.g., Venter and Haftka¹¹ 1997).

An approach where the response surface approximation is built from data points selected by the optimizer has also recently been used in structural optimization. For example, Harrison, Le Riche and Haftka¹² (1995) and Sellar and Batill¹³ (1996) used the approach to construct response surface approximations that are periodically updated throughout the design process. The response surface approximation is updated by adding new design points,

generated by the optimizer, to the set of design points used for constructing the response surface approximation. Wujek et al.¹⁴ (1996) and Rodríguez, Renaud and Watson¹⁵ (1997) applied the approach to multidisciplinary design optimization, constructing local response surface approximations in a sequential manner based on design points evaluated at the discipline level of the current design cycle. Additionally, Cramer and Carter¹⁶ (1997) considered design points generated by a global optimization algorithm. A statistical analysis of these design points was then used to complement the global optimization process by giving more insight into the system behavior within the design space. This additional information was then used to influence the design process, for example to eliminate design variables that have a small influence on the system response.

1.3. Response Surface Approximation Applications in Optimization

As discussed in the previous section, the problems associated with the direct use of detailed numerical simulations in an optimization environment may be alleviated by the use of approximation concepts. When using approximation concepts, the computational burden is shifted from the optimization problem to the problem of constructing the approximations. A detailed discussion of basic approximation concepts in structural optimization is presented by Barthelemy and Haftka¹⁷ (1993).

It has long been recognized in the field of structural optimization that, compared to some of the direct methods, for example the method of feasible directions (e.g., Haftka and Gürdal¹⁸ 1993, pp. 182-186), approximation concepts are an effective approach for reducing the computational cost associated with optimization problems. Schmit and his co-workers (e.g., Schmit and Farshi¹⁹ 1974 and Schmit and Miura²⁰ 1976) demonstrated first that, when applied correctly, approximation concepts might significantly improve the efficiency of

solving structural optimization problems. The traditional approach is to use local derivative-based approximations (typically some variation of the Taylor series expansion) that are generally applied sequentially, with new approximations constructed at each new design cycle. These local derivative-based approximations are constructed from information obtained at a single design point, and help to reduce the magnitude of the problem areas associated with the use of detailed numerical analyses in an optimization environment. Unfortunately, as a result of their local nature and sequential application, the problem areas are not eliminated. The designer still lacks a global perspective of the response, human error and numerical noise may still cause the optimization algorithm to converge on spurious local optima, it is still difficult to take full advantage of parallel computing techniques, and interaction between different analysis codes is still required.

Multi-point approximations are a variation of the traditional local derivative-based approximations that utilize both function and derivative information from several design points in constructing an approximation. Compared to traditional derivative-based approximations, typically constructed from function and derivative information obtained from a single design point, multi-point approximations are generally more accurate and global in nature. Haftka et al.²¹ (1989) examined approximations based on two and three design points and concluded that the approximations worked well for interpolation, while only marginal improvements in accuracy were obtained when extrapolating. Fadel et al.²² (1990) introduced a two point exponential approximation that uses derivative information from the previous design point, while Snyman and Stander²³ (1994) used only the previous function value in creating a two point quadratic approximation. Additionally, Wang and Grandhi^{24,25} (1994 and 1995) introduced two point approximations that use both function and derivative

information from a previous design point. Wang, Grandhi and Hopkins²⁶ (1994), Wang, Grandhi and Canfield²⁷ (1994) and Wang and Grandhi²⁸ (1996) introduced several other multi-point approximations, using information from more than two design points. These approximations were applied successfully to various truss, frame, shape, and reliability-based optimization problems. In their work, the set of design points used for constructing the approximation is expanded at the end of each design cycle by performing an accurate analysis of the current optimum design. As the design process proceeds more design points are used for constructing the approximation, thus gradually improving the accuracy and expanding the region of applicability.

More recently, response surface approximations have been used extensively for obtaining multi-point approximations. Compared to traditional multi-point approximations as discussed in the previous paragraph, the multi-point approximations based on response surface methodology do not have to interpolate between the design points but use a least squares procedure. Multi-point approximations based on response surface methodology differ from standard response surface approximations, since they are constructed from design points selected by the optimizer. In contrast, the standard response surface approximations are constructed from data points obtained from statistical design of experiments. Additionally, some of these multi-point approximations make use of derivative information in constructing the approximations (e.g., Sellar and Batill¹³ 1996 and Rodríguez, Renaud and Watson^{15, 29} 1997) while standard response surface approximations use only function values.

In the literature, multi-point approximations based on response surface methodology are implemented in many different formulations. For example, Toropov, Filatov and Polynkin³⁰ (1993) used response surface methodology to develop a multi-point approximation

design methodology. The approach employs move limits during the optimization process to define a sub-region about the current optimum design point. A weighted least squares procedure is then used to formulate an explicit local approximation near the current design point, according to the move limit strategy. Both new design points evaluated within the current sub-region as well as several design points evaluated during previous design cycles are used to construct the approximations. The methodology is applied successfully to several example problems, e.g., truss and beam structures (Toropov, Filatov and Polynkin³⁰ 1993), performance optimization of small Stirling engines (Toropov, Markin and Carlsen³¹ 1994 and Toropov and Carlsen³² 1994) and the optimization of geometrically nonlinear shell structures (Van Keulen, Toropov and Polynkin³³ 1994).

Sellar and Batill¹³ (1996) also constructed multi-point approximations based on response surface methodology, but used a slightly different implementation specifically tailored to a multidisciplinary optimization environment. At each new design cycle, function and derivative information obtained from design points identified by the optimizer were added to the database used for constructing the response surface approximation. The response surface approximation is thus updated at each new design cycle with new information obtained from design points identified by the optimizer. Harrison, Le Riche and Haftka¹² (1995) used a similar approach. They made use of a genetic algorithm to minimize the weight of a hat stiffened composite panel and constructed a response surface approximation based on design points identified by the genetic algorithm. Their response surface approximation is updated periodically, after a fixed number of iterations are completed by adding new design points identified by the genetic algorithm to the set of design points used for constructing the response surface approximation. In their approach, the most recent design points were given a

higher weight as compared to the older design points, and the most recent design points thus have a larger influence on the resulting response surface approximation.

Venkataraman and Haftka¹⁴ (1997) and Venter, Haftka and Chirehdast³⁵ (1997) used local response surface approximations constructed in a sequential manner, in single discipline design optimization. After each design cycle, a new response surface approximation is constructed using statistical design of experiments to select data points for constructing the response surface approximation near the current optimum design. Wujek et al.¹⁴ (1996) and Rodríguez, Renaud and Watson¹⁵ (1997) considered a similar approach in multidisciplinary design optimization problems, but used design points evaluated at the discipline level during the current design cycle for constructing the sequential response surface approximations. Rodríguez, Renaud and Watson¹⁵ (1997) made use of trust region constraints to define a sub-region near the current optimum design. At each design cycle, a new response surface approximation is constructed for the defined trust region. These approximations are constructed so that the function and the first derivative match the actual values at the current design point exactly, and thus only the second order parameters are obtained using a least squares procedure. Rodríguez, Renaud and Watson²⁹ (1997) constructed similar approximations, but used statistical design of experiments to select the data points for constructing the response surface approximations.

All of the above response surface approximation formulations have the advantage of not requiring derivative information when constructing the approximations. However, the use of derivative information is permitted and was used in some cases (e.g., Ref. 13, 15 and 29). Additionally, these approximations have a tendency to eliminate spurious local minima resulting from human and numerical error, while some of the formulations (Ref. 12 and 13)

result in approximations that are more general in nature when compared to the traditional local derivative-based approximations. Although these response surface approximations are successful in alleviating the problem areas associated with using detailed numerical analyses in an optimization environment, the problem areas are not completely eliminated due to the fact that these approximations still do not provide global approximations that are valid over the entire design space.

When using polynomial response surface approximations that are global in nature, however, the problems associated with using modern analysis procedures in an optimization environment may be eliminated. Apart from providing the designer with a global perspective of the response, spurious local minima resulting from human and numerical errors are eliminated by using the least squares method to formulate smooth low order polynomials. These approximations are constructed independently and no interaction between different software codes is required. In many cases the computational cost of structural optimization problems may be reduced by using response surface approximations that are global in nature, since the number of function evaluations required to construct these response surface approximations may be less than that needed to perform the optimization. Finally, the use of parallel computing techniques is facilitated in that the response surface approximations are constructed from independent data points, which may be evaluated in parallel with very little communication between the different processors.

In recent years, many researchers have exploited the advantages of using standard response surface approximations that are more global in nature to integrate numerical analyses in an optimization environment. With few exceptions (e.g., Van Houten et al.³⁶ 1995), almost all the standard applications of response surface approximations in structural optimization

employ quadratic approximations which still do not provide fully global approximations. For example, Thacker and Wu³⁷ (1992) used response surface approximations to reduce the computational cost associated with reliability-based optimization problems using probabilistic methods, while Harrison, Le Riche and Haftka¹² (1995) reduced the cost of designing a stiffened composite panel using a genetic algorithm. Van Houten et al.³⁶ (1995), Giunta et al.³⁸ (1996), Kaufman et al.³⁹ (1996) and Venter, Haftka and Chirehdast³⁵ (1997) showed that response surface approximations are valuable in solving optimization problems with non-smooth functional behavior, since numerical noise is filtered out from the resulting approximate response function. Additionally, Mistree et al.⁴⁰ (1994) made extensive use of the global perspective of the response over the design space provided by global response surface approximations, while Brown and Nachlas⁴¹ (1985), White et al.⁴² (1985) and Giunta et al.⁴³ (1995) used response surface approximations to meet the organizational challenge posed by multidisciplinary optimization problems. More specifically, Mason et al.⁴⁴ (1994) used response surface approximations to integrate the analysis and optimization codes in a single discipline optimization, while Ragon et al.⁴⁵ (1997) used response surface approximations as a simple, yet flexible, interface between the global and local design codes in the design of a large wing structure.

The Multidisciplinary Analysis and Design (MAD) Center of Virginia Tech considered the aerodynamic-structural optimization of the High Speed Civil Transport (HSCT) and made extensive use of the fact that response surface approximations facilitate the use of parallel computing techniques (e.g., Giunta et al.⁴³ 1995, Balabanov et al.¹⁰ 1996, Burgee et al.⁴⁶ 1996 and Kaufman et al.³⁹ 1996). They made use of an Intel Paragon machine with more than 100 processors to perform a large number of structural optimizations and showed a

substantial reduction in real time by performing these optimizations in parallel. The data obtained from these optimizations were used to construct response surface approximations for the wing bending material weight of the High Speed Civil Transport.

Unfortunately, response surface approximations are plagued by what is known as the curse of dimensionality. The curse of dimensionality implies that the number of data points required to construct a response surface approximation increases greatly when more design variables are added to the response surface approximation. A substantial reduction in the cost of constructing a response surface approximation, often equal to a few orders of magnitude, can be realized by reducing the number of design variables in the response surface approximation. Identifying the variables that are intrinsic to the problem under consideration, and using these intrinsic variables as design variables in a response surface approximation is an effective approach for reducing the number of design variables in a response surface approximation. The use of intrinsic variables has the additional advantage of increasing the accuracy of the resulting response surface approximation (e.g., Kaufman et al.¹ 1996 and Roux, Stander and Haftka⁴⁷ 1996).

1.4. Dimensional Analysis

Dimensional analysis provides a methodology for identifying the intrinsic variables of a system by formulating the governing equations of the system in non-dimensional form. This technique is discussed in detail in Barenblatt⁴⁸ (1996, pp. 1-63). Dimensional analysis is based on the simple idea that any physical law is independent of the arbitrary chosen units of measurement used for describing a specific system. Instead, physical laws possess a fundamental property that may be expressed in terms of variables intrinsic to the system. The use of dimensional analysis as a general methodology for identifying the intrinsic variables of a

system is justified by the Buckingham Π -Theorem (Buckingham⁴⁹ 1915), which may be stated as:

A physical relationship between some dimensional (generally speaking) quantity and several dimensional governing variables can be rewritten as a relationship between some dimensionless quantity and several dimensionless products of the governing variables; the number of dimensionless products is equal to the total number of governing variables minus the number of governing variables with independent dimensions.

The governing variables (also referred to as predictor variables in this dissertation) refer to the dimensional variables used in describing the system. An independent dimension is defined as a dimension that cannot be expressed in terms of a product of powers of the dimensions of the remaining variables.

By using the non-dimensional intrinsic variables obtained from the non-dimensional formulation of the governing equations, the number of design variables in a response surface approximation may be greatly reduced. Additionally, a non-dimensional formulation of the governing equations of a system results in more general solutions that are applicable to a wider range of problems. A good example is the stress concentration design charts published by Peterson⁵⁰ (1953). These charts are applicable to any isotropic and homogeneous material and are independent of the absolute geometry and loading conditions of the structure under consideration.

1.5. Fuzzy Set Based Design Optimization

A typical example of an expensive structural optimization problem that will be considered in the present dissertation is designing for uncertainty. Designing of uncertainty often occurs in the aircraft industry, where structures are designed that will be built well into the future from materials available then. Oftentimes, these materials are not yet available, forcing the designer to deal with uncertain material properties. For this type of design problem, little information regarding the uncertainty is known and the uncertainty is typically based on expert opinion and assumptions made by the designer. Unlike probabilistic methods, which require a large amount of data for modeling the uncertainty, fuzzy set theory is capable of modeling uncertainty using the limited data available to the designer. Apart from requiring a large amount of data, the results obtained from probabilistic methods are, in some cases, very sensitive to the accuracy of the data as well as to assumptions made during the modeling process (Ben-Haim and Elishakoff⁵¹ 1990, pp. 11-32). In contrast, fuzzy set theory caters to worst case scenarios, taking account of the fact that the uncertainty is modeled based on subjective opinions and assumptions (Maglaras et al.⁵² 1997).

Fuzzy set theory was introduced by Zadeh⁵³ (1965) as a mathematical tool for the quantitative modeling of uncertainty. Fuzzy set theory makes use of fuzzy numbers to represent uncertain problem parameters. In order to represent an uncertain parameter as a fuzzy number, the designer only needs to specify the range of uncertainty and a membership function that denotes the possibility of occurrence of an element in the specified range.

In recent years, fuzzy set theory has been applied to a wide range of structural optimization problems. For example, Liu and Huang⁵⁴ (1992) performed a fatigue reliability analysis of a portal frame, Jung and Pulmano⁵⁵ (1996) considered the optimal plastic design of a

fixed-fixed beam and a portal frame, and Jensen and Sepulveda⁵⁶ (1997) minimized the weight of a 25 bar transmission tower. Fuzzy set theory has also been used in multidisciplinary optimization by Rao⁵⁷ (1993), who designed the main rotor of a helicopter as well as by Wu and Young⁵⁸ (1996) who optimized the machine room layout of a ship. Additionally, Shih and Chang⁵⁹ (1995) applied multi-criteria optimization to various truss examples by considering both weight and displacement as objectives.

Unfortunately, designing for uncertainty is computationally intensive, typically requiring an order of magnitude more function evaluations than a corresponding deterministic design problem. In reliability optimization using probabilistic methods, approximation concepts have been widely used to reduce the computational cost e.g., Torng and Thacker⁶⁰ (1993), Wang, Grandhi and Hopkins²⁶ (1994), Sepulveda and Jensen⁶¹ (1995), Wang and Grandhi^{28, 62} (1996), Grandhi and Wang⁶³ (1996) all used various approximation concepts, although none of them considered response surface approximations. However, Thacker and Wu³⁷ (1992) used response surface approximations in a probabilistic-based reliability analysis of a turbine blade subject to creep rupture, while Fox⁶⁴ (1996) investigated various aspects associated with using response surface approximations with accurate probabilistic design.

Compared to probabilistic methods, the use of fuzzy set theory to model uncertainty in structural optimization problems, is a more recent development. Sepulveda and Jensen⁶⁵ (1996) and Jensen and Sepulveda⁵⁶ (1997) reduced the computational cost associated with performing fuzzy set based optimization by using linear Taylor series expansions to form local approximations that are applied in a sequential manner. In fuzzy set theory, the high computational cost is associated with the evaluation of the possibility of failure, which requires multiple evaluations of a global optimization problem that is defined in terms of the

uncertainty problem parameters. Additionally, the possibility of failure has to be evaluated many times for each design cycle. In terms of fuzzy set based optimization, response surface approximations have the advantage that the number of function evaluations required for constructing response surface approximations of the required analysis procedures, are typically less than the number of function evaluations required to perform a single design cycle. Furthermore, once the response surface approximations are constructed they may be used repeatedly without requiring any new function evaluations, thus allowing multiple optimizations at minimal additional computational cost.

In the present dissertation, the possibility of failure is calculated numerically by making use of the vertex method (Dong and Shah⁶⁶ 1987), which is based on the definition of a fuzzy function. The cost of evaluating the possibility of failure is reduced by replacing costly finite element analyses with previously developed response surface approximations. A response surface approximation of the possibility of failure is constructed from numerical simulations, which involves the calculation of the possibility of failure for different combinations of the design variables. The possibility of failure response surface approximation is thus constructed based on results obtained from the response surface approximations constructed from the finite element data. The possibility of failure response surface approximation simplifies the integration of the analysis code and the optimization algorithm and filters out numerical noise in the approximate response function, thus enabling the use of a derivative-based optimization algorithm.

1.6. Objectives of Dissertation

The first objective of the present dissertation is to investigate the use of response surface approximations in expensive structural optimization problems, and to develop an

efficient methodology for constructing highly accurate response surface approximations for use in structural optimization applications. Even though the use of response surface approximations in design optimization has several advantages, the construction of response surface approximations is not a trivial process and may be computationally expensive in itself. Furthermore, response surface approximations are a fairly new technology in the area of structural optimization, and a need exists for an efficient methodology, tailored specifically to the field of structural optimization. The methodology discussed in the present dissertation consists of a combination of both existing techniques not commonly used in response surface methodology (e.g., dimensional analysis and higher order polynomials as response surface approximations) as well as some existing response surface methodology techniques not often used in structural optimization applications (e.g., stepwise regression and detailed error analysis).

The second objective of this dissertation is to illustrate the advantages of using response surface approximations in expensive structural optimization problems, by considering designing for uncertainty. A stepped plate design problem is considered as an illustrative example, and response surface approximations are constructed for evaluating different failure criteria of the plate using detailed numerical simulations conducted with finite element analyses. Both an isotropic as well as a composite laminated plate, where the change in thickness is a result of an internal ply drop off, are considered. The problem parameters are uncertain and the uncertainty is modeled by means of fuzzy set theory. Response surface approximations are used: (1) to reduce the computational cost of designing for uncertainty, (2) to integrate the analysis code and the optimization algorithm, and (3) to filter out numerical noise inherent to the response function.

1.7. Outline of Dissertation

Response surface methodology, which is a key aspect of the present dissertation, is reviewed in *Chapter 2*. The review concentrates on aspects of response surface methodology that are relevant to the present dissertation and the use of response surface approximations in structural optimization applications in general.

Chapter 3 introduces fuzzy sets, fuzzy numbers and fuzzy functions and describes the relationship between the membership function of a fuzzy number and the possibility distribution function. Additionally, the vertex method is introduced as a numerical means of calculating membership functions for fuzzy functions (a function of fuzzy numbers).

In *Chapter 4* an isotropic stepped plate example problem and its associated response surface approximations are introduced and discussed in detail, while *Chapter 5* summarizes the optimization results obtained. Similarly, in *Chapter 6* a dropped ply composite laminated plate and its associated response surface approximations are introduced in discussed in detail, with the corresponding optimization results summarized in *Chapter 7*.

Finally, concluding remarks are outlined in *Chapter 8*.

CHAPTER 2

RESPONSE SURFACE METHODOLOGY

Response surface methodology may be defined as “a collection of statistical and mathematical techniques useful for developing, improving and optimizing processes” (Myers and Montgomery⁶ 1995, p. 1) or as “a group of statistical techniques for empirical model building and model exploration” (Box and Draper⁷ 1987, p. 1). The methodology was originally developed for constructing empirical response functions, known as response surface approximations or response models, based on a set of physical experiments. In the present dissertation, the physical experiments are replaced by numerical experiments in the form of finite element analyses. Response surface methodology typically involves the identification of a set of experimental data points at which to evaluate the response function, the construction of an approximate response function based on the experimentally obtained data, and the evaluation of the predictive capabilities of the approximate response function. Finally, the resulting response surface approximations are used in optimization.

2.1. Response Surface Approximation Construction

A response surface approximations is an approximate relationship between a response η (the dependent variable) and a vector $\mathbf{z}$ of k predictor variables (the independent variables). The response is generally obtained from experiments (possibly numerical in nature) in which case η denotes the mean or expected response value. The experimentally obtained response y differs from the expected value η due to random experimental error δ . The relation between y and the predictor variables may be written as

$$y = \eta + \delta = F(z_1, z_2, \dots, z_k) + \delta \quad (2.1)$$

where F is the exact dependence of η on $\mathbf{z}$. If a response surface approximation f is used to approximate F , Eq. (2.1) may be written as

$$y = f(z_1, z_2, \dots, z_k) + \varepsilon \quad (2.2)$$

where ε denotes the total error, which is the difference between the predicted and the expected response values, and includes both the random (variance) error δ and the modeling (bias) error $(F - f)$. The response surface approximating f is generally chosen to be a low order polynomial, and the predicted response vector $\hat{\mathbf{y}}$ may then be written in matrix form as

$$\hat{\mathbf{y}} = \mathbf{X}\mathbf{b} \quad \text{or} \quad \begin{Bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{Bmatrix} = \begin{bmatrix} 1 & x_{12} & x_{13} & \cdots & x_{1p} \\ 1 & x_{22} & x_{23} & \cdots & x_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n2} & x_{n3} & \cdots & x_{np} \end{bmatrix} \begin{Bmatrix} b_1 \\ b_2 \\ \vdots \\ b_p \end{Bmatrix} \quad (2.3)$$

where the caret symbol denotes predicted values. In Eq. (2.3), n denotes the number of data points considered, and p denotes the number of parameters in the response surface approximation. Furthermore, $\mathbf{X}$ is a matrix of response surface approximation parameters (usually monomials) evaluated at the data points, and the vector $\mathbf{b}$ contains estimates of the unknown coefficients β_i of the response surface approximation f .

The unknown coefficients β_i of the response surface approximation are generally estimated using the least squares procedure (e.g., Burden and Faires² 1989, pp. 427-439). The least squares procedure calculates estimates b_i of the actual coefficients β_i in such a way as to minimize the sum L of the squares of the error terms as follows:

$$\left. \frac{\partial L}{\partial \beta_i} \right|_{b_i} = -2\mathbf{X}^T \mathbf{y} + 2\mathbf{X}^T \mathbf{X} \mathbf{b} = 0 \quad \text{where} \quad L = \sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (y_i - f(z_1, z_2, \dots, z_k))^2 \quad (2.4)$$

The estimates of the regression coefficients are solved from Eq. (2.4), which may be rewritten as follows:

$$\mathbf{b} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y} \quad (2.5)$$

The difference between the predicted response values obtained from the response surface approximation and the response values obtained from the experiments is known as the residual error, defined for the i^{th} data point as:

$$e_i = y_i - \hat{y}_i \quad (2.6)$$

2.2. Statistical Design of Experiments

Statistical design of experiments is an integral part of response surface methodology and enables the designer to construct response surface approximations more efficiently by seeking a small set of optimal data points at which to evaluate the response function. This optimal set of data points is chosen to maximize the accuracy by minimizing either the variance or the bias error associated with the resulting response surface approximation.

In cases where the variance error is dominant, for example when the assumed response surface approximation is sufficient to accurately approximate the actual response function but the data are subject to experimental error, it is appropriate to find a set of optimal data points by minimizing the variance error. For regularly shaped design spaces there are a number of standard designs (i.e., sets of data points) available that minimize the variance error, for example the Central Composite (Box and Wilson⁸ 1951) and the Box-Behnken (Box and Behnken⁹ 1960) designs. For an irregularly shaped design space, the designer must resort to one of several possible computer generated designs (e.g., Myers and Montgomery⁶ 1995, pp. 364-366 and Box and Draper⁷ 1987, pp. 489-501), of which the best known and most widely used is the *D*-Optimality design criterion. The *D*-Optimality design criterion

minimizes a measure of the variance associated with the estimates of the coefficients of the response surface approximation, by maximizing the determinant of the $\mathbf{X}^T\mathbf{X}$ matrix. When assuming that the errors e_i of the response surface approximation are independent and have a normal distribution with zero mean and constant variance, the determinant of the $\mathbf{X}^T\mathbf{X}$ matrix is inversely proportional to the square of the volume of the confidence region associated with the regression coefficients $\mathbf{b}$. The volume of the confidence region provides an indication of how well the coefficients of the response surface approximation are estimated, with a smaller volume corresponding to better estimation. The D -Optimality criterion thus selects a set of data points from a candidate set of data points to maximize the determinant of the $\mathbf{X}^T\mathbf{X}$ matrix, and the best D -Optimal design is obtained from

$$\text{Max}_{\zeta} \left(\frac{\text{Det}(\mathbf{X}^T\mathbf{X})}{n^p} \right) \quad (2.7)$$

where ζ represents all designs in the set of possible designs. It is important to note that variance-based design criteria assume that all of the error associated with the response surface approximation is random, and do not address bias or modeling errors resulting from an insufficient response surface approximation.

In cases where the bias error is dominant, for example when low order polynomials are used to approximate a response quantity with highly non-linear behavior, it may be more appropriate to find a set of optimal data points by minimizing the bias error rather than the variance error. For a regularly shaped design space and a fixed number of data points, closed form solutions exist for finding the minimum-bias data points (e.g., Myers and Montgomery⁶ 1995, pp. 406-421 and Box and Draper⁷ 1987, pp. 437-442). However, for an irregularly shaped design space no closed form solution exists and the minimum-bias data points must be obtained numerically (e.g., Venter and Haftka¹¹ 1997).

Even though minimum-bias statistical designed experiments may be more appropriate for some response surface approximations, virtually all applications within the engineering optimization literature make successful use of some variance-based statistical design of experiments. In the present dissertation where relatively high order polynomials (cubic and quartic) are used as response surface approximations, it was found that the predictive capabilities of the response surface approximations were not significantly improved by using a minimum-bias design criterion. Additionally, it is difficult to implement a minimum-bias design for an irregular design space with a large number of predictor variables, and it was thus decided to use the *D*-Optimality criterion whenever statistical design of experiments was required. In all cases, the *D*-Optimality algorithm provided with the JMP⁶⁷ (1995) software package was used.

2.3. Elimination of Redundant Parameters

Since the exact form of the actual response function is rarely known, polynomial functions are generally assumed as response surface approximations. Due to the general nature of such a polynomial response function, the response surface approximation will usually include redundant parameters, or parameters poorly characterized by the experiments. These parameters may increase the error of the response surface approximation, thus decreasing its predictive capabilities (Myers and Montgomery⁶ 1995, p. 641, and Kaufman et al.³⁹ 1996). Eliminating these parameters from the response surface approximation is an important step in constructing highly accurate response surface approximations.

Several methods exist for selecting the best subset of parameters from the general polynomial response function to be used as a final response surface approximation. Examples of the most popular procedures are the so called “all-possible-regressions” and the “stepwise

regression" procedures (e.g., Myers and Montgomery⁶ 1995, pp. 642-655 and Ott⁶⁸ 1993, pp. 648-659). The all-possible-regressions procedure requires that all possible combinations of parameters from the general response surface approximation be examined. The best subset of parameters is then chosen based on a suitable criterion. The procedure requires the examination of 2^p partial response surface approximations for p parameters in the general response surface approximation.

The stepwise regression procedure was developed to provide an alternative to the high computational cost associated with the all-possible-regressions procedure, especially for response surface approximations with a large number of parameters. The stepwise regression procedure represents a family of procedures that examine only a small number of partial response surface approximations by either adding or deleting one parameter at a time and may be classified as either a forward, a backward, or a mixed stepwise regression procedure. The forward stepwise regression procedure assumes a response surface approximation with only one parameter, the intercept. Parameters are then added one at a time by selecting the parameter that results in the largest increase in predictive capabilities of the current response surface approximation. This process of adding parameters is repeated until a specified termination criterion is satisfied. In contrast, the backward stepwise regression procedure starts with a response surface approximation that includes all the parameters of the general response surface approximation. From this response surface approximation, one parameter is deleted at a time, selecting the parameter with the minimum influence on the predictive capabilities of the resulting response surface approximation. Finally, the mixed stepwise regression procedure consists of a combination of both the forward and backward stepwise regression procedures.

It is important to note that stepwise regression procedures do not guarantee the best subset of parameters and must be used with caution. However, good results were obtained by using the mixed stepwise regression procedure provided with the JMP⁶⁷ (1995) software package. This procedure starts with an initial response surface approximation that includes all the parameters of the general response surface approximation and then applies backward stepwise regression. At each step of the backward stepwise regression, the least significant parameter is removed from the current response surface approximation. Additionally, at each step all removed parameters are re-examined for possible inclusion in the response surface approximation. The procedure may be summarized as:

1. Start with the full response surface approximation as the initial response surface approximation.
2. Apply backward stepwise regression, thus deleting the least significant parameter from the current response surface approximation.
3. Apply forward stepwise regression to all of the deleted parameters. This procedure evaluates if any of the deleted parameters has a significant influence on the predictive capability of the current response surface approximation. The criterion used to determine when to include a parameter is generally more stringent than the criterion used to remove a parameter.
4. Go to step 2 and repeat until some termination criterion is satisfied.

In the present dissertation, Mallows's C_p statistic was used as the termination criterion to identify the best reduced response surface approximation from the subset of reduced response surface approximations provided by the stepwise regression procedure. The C_p

statistic addresses the prediction capabilities of the response surface approximation by estimating a total error. This total error consists of two parts, one resulting from modeling error (bias) and the other from noise (variance error). An under specified response surface approximation (not enough parameters) increases that part of the total error resulting from bias, while an over specified response surface approximation (too many parameters) increases that part of the total error resulting from variance (Ott⁶⁸ 1993, p. 653). The C_p statistic is defined as

$$C_p = \left(\frac{SSE_p}{MSE} \right) - (n - 2p) \quad (2.8)$$

where SSE_p is the sum of the squares of the n errors for a response surface approximation with p parameters (including the intercept or constant parameter) and MSE is the mean sum of squares of the error terms obtained from the response surface approximation with all parameters included. Based on the C_p statistic, the best response surface approximation in a set of candidate response surface approximations is the response surface approximation with the lowest C_p value, and a good response surface approximation should at least have a C_p value close to p .

2.4. Evaluation of Predictive Capabilities

Optimization has the general tendency of exploiting weaknesses in the formulation of the response function. In terms of response surface approximations, this means that the optimizer would tend to drive the optimum design to a region where the approximations are inaccurate. Highly accurate response surface approximations are thus a requirement for structural optimization applications, and it is important to evaluate the predictive capabilities of the approximations. In the present dissertation, a detailed error analysis using two data sets

and various statistical criteria were used to evaluate the predictive capabilities of the response surface approximations. Data Set 1 contains all data points used in constructing the response surface approximations, while Data Set 2 is an independent data set.

For Data Set 1, the coefficient of determination (R^2) statistic, the adjusted R^2 ($Adj-R^2$) statistic, the percent root mean square error ($\%RMSE$), and the percent root mean square error based on the predicted sum of squares ($PRESS$) statistic ($\%RMSE_{PRESS}$) are calculated (Myers and Montgomery⁶ 1995, pp. 28-47). The coefficient of determination (R^2) statistic is the proportion of the variability in the response data that is accounted for by the response surface approximation and is defined as

$$R^2 = 1 - \frac{SSE}{S_{yy}} \quad (2.9)$$

$$\text{where} \quad SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad \text{and} \quad S_{yy} = \sum_{i=1}^n y_i^2 - \frac{\left(\sum_{i=1}^n y_i \right)^2}{n} \quad (2.10)$$

The coefficient of determination has a value between 0 and 1. Unfortunately, a value close to 1 does not necessarily imply a good response surface approximation because additional parameters added to a response surface approximation will always increase the R^2 value without necessarily increasing the predictive capabilities of the response surface approximation. The adjusted coefficient of determination ($Adj-R^2$) statistic is an alternative measure of the explained variability that is often used and has the desirable property that its value does not necessarily increase when adding additional parameters to a response surface approximation. When a large difference exists between the R^2 and $Adj-R^2$ values, there exists a good chance that the response surface approximation contains insignificant parameters. The $Adj-R^2$ statistic is defined as:

$$Adj-R^2 = 1 - \frac{SSE/(n-p)}{S_{yy}/(n-1)} = 1 - \left(\frac{n-1}{n-p} \right) (1-R^2) \quad (2.11)$$

The percentage errors are defined in relation to the average absolute value $\bar{y}$ of the measured response data, where

$$\bar{y} = \frac{1}{n} \sum_{i=1}^n |y_i| \quad (2.12)$$

and the %RMSE is defined as

$$\%RMSE = \frac{100}{\bar{y}} \sqrt{\frac{1}{\chi} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (2.13)$$

where $\chi = (n-p)$ for Data Set 1, and $\chi = n$ for Data Set 2. For Data Set 1, the $(n-p)$ factor is used to obtain an unbiased estimator of the mean square error of the response surface approximation, thus compensating for the fact that the error is calculated at the data points used to construct the response surface approximation. Even though Eq. (2.13) is based on an unbiased estimator of the mean square error, the %RMSE value may be overly optimistic in estimating the predictive capabilities of the response surface approximation. A more realistic estimate of the predictive capabilities of a response surface approximation is obtained when defining the %RMSE based on the *PRESS* statistic. The *PRESS* statistic is calculated by selecting a data point, say data point i . The response surface approximation obtained from the remaining $(n-1)$ data points is then used to predict the response at the withheld data point, denoted by $\hat{y}_{(i)}$. The prediction error at the withheld data point $e_{(i)}$ may then be written as

$$e_{(i)} = y_i - \hat{y}_{(i)} \quad (2.14)$$

and is referred to as the i^{th} *PRESS* residual. This procedure is then repeated for all data points and the resulting *PRESS* residuals are summed to obtain the *PRESS* statistic as follows:

$$PRESS = \sum_{i=1}^n e_{(i)}^2 = \sum_{i=1}^n (y_i - \hat{y}_{(i)})^2 \quad (2.15)$$

However, from (Myers and Montgomery⁶ 1995, p. 43-47) the i^{th} *PRESS* residual may also be written as

$$e_{(i)} = \frac{e_i}{1 - h_{ii}} \quad (2.16)$$

where h_{ii} denotes the diagonal elements of the matrix $\mathbf{H}$, which is define by:

$$\mathbf{H} = \mathbf{X}(\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \quad (2.17)$$

The formulation of $e_{(i)}$ shown in Eq. (2.16) is significant since it allows the calculation of the *PRESS* statistic from a single linear regression, by using the relationship:

$$PRESS = \sum_{i=1}^n \left(\frac{e_i}{1 - h_{ii}} \right)^2 \quad (2.18)$$

The %*RMSE* based on the *PRESS* statistic (%*RMSE_{PRESS}*) is then defined as:

$$\%RMSE_{PRESS} = \frac{100}{\bar{y}} \sqrt{\frac{1}{n} PRESS} \quad (2.19)$$

Apart from the %*RMSE*, the percent average error (%*AvgErr*) and the percent maximum error (%*MaxErr*) are also calculated for Data Set 2 as follows:

$$\%AvgErr = \frac{100}{n\bar{y}} \sum_{i=1}^n |y_i - \hat{y}_i| \quad (2.20)$$

$$\%MaxErr = \text{Max}_i \left[\frac{100}{\bar{y}} |y_i - \hat{y}_i| \right] \quad (2.21)$$

2.5. Outliers, Leverage and Influential Data Points

Apart from evaluating the predictive capabilities of the constructed response surface approximations, it is also important to evaluate the quality of the data points used to construct the response surface approximations. An evaluation of the data points is required since it is

possible for a small set of data points, known as influential data points, to have a disproportionately high influence on the resulting response surface approximation. An influential data point is defined as a data point which, if removed, the resulting response surface approximation changes considerably. Influential data points are typically data points with high leverage and an outlier tendency.

The leverage of a data point is related to the location of that data point relative to the centroid of all data points used to construct the response surface approximation. Data points located far from the centroid have high leverage. The diagonal elements h_{ii} of the matrix **H** Eq. (2.17) are useful in determining the leverage of data points. The diagonal elements h_{ii} are bounded by $0 < h_{ii} < 1$ with an average value of p/n . According to Myers and Montgomery⁶ (1995, p. 49) a rough guideline is that a h_{ii} value greater than $2p/n$ would indicate that the i^{th} data point is a high leverage data point.

An outlier is a data point with a large residual error e_i and may be a result of either inaccurate experimental data or an inadequate response surface approximation. Outliers are typically detected by using the *R*-Student residual statistic t_i , which is obtained by scaling the residual e_i of the i^{th} data point by (Myers and Montgomery⁶ 1995, pp. 43-48)

$$t_i = \frac{e_i}{\sqrt{s_{(i)}^2(1-h_{ii})}} \quad \text{and} \quad s_{(i)}^2 = \frac{(n-p)MSE - e_i^2/(1-h_{ii})}{n-p-1} \quad (2.22)$$

where $s_{(i)}$ is an estimate of the variance of the error terms of the data set with the i^{th} data point removed. Compared to the unscaled residuals, the *R*-Student residuals have constant variance $\text{Var}(t_i)=1$ regardless of the position of the i^{th} data point and are more sensitive to influential data points. Any t_i value outside the interval $-3 \leq t_i \leq 3$ should be considered as possible outliers.

Influential data points are typically difficult to detect since their residuals may be artificially small due to the large influence these data points have on the resulting response surface approximation. The Cook's D statistic is used to identify influential data points and is defined as (Myers and Montgomery⁶ 1995, pp. 49-50):

$$D_i = \frac{(\mathbf{b}_{(i)} - \mathbf{b})^T \mathbf{X}^T \mathbf{X} (\mathbf{b}_{(i)} - \mathbf{b})}{p \text{MSE}} \quad (2.23)$$

The Cook's D statistic is a measure of the squared distance between the vector of least squares estimated coefficients $\mathbf{b}$ obtained from all of the n data points and the vector of least squares estimated coefficients $\mathbf{b}_{(i)}$ obtained by deleting the i^{th} data point from the original set of data points. According to Myers and Montgomery⁶ (1995, pp. 49-50), data points with a Cook's D statistic larger than 1 is considered as influential data points. Influential data points may also be detected by using robust regression procedures which are designed to dampen the effect that influential data points may have on a response surface approximation obtained from the normal least squares procedure. Robust regression is typically applied using an iteratively reweighted least squares procedure that assigns weights to all data points used to construct the response surface approximation. Influential data points are assigned a low weight, thus increasing the associated residuals (as compared to those obtained from the normal least squares procedure) and making it much easier to identify influential data points. A detailed discussion of robust regression techniques are beyond the scope of this dissertation and the interested reader is referred to Myers and Montgomery⁶ (1995, pp. 667-673) and Box and Draper⁷(1987, pp. 85-90) as an introduction with several additional references. When influential data points are detected, these data points should be carefully investigated to determine if they are influential as a result of inaccurate experimental data or due to an inadequate response surface approximation, before taking appropriate action.

In order to evaluate the need for an extensive investigation to identify influential data points in the data sets of the present dissertation, a few representative sample data sets were investigated first. These data sets were investigated using the *R*-Student and Cook's *D* statistics as well as an iteratively reweighted least squares procedure to ensure that the data points were of good quality. It was found that since accurate finite element analyses and appropriate high order polynomials were used in constructing the response surface approximations, no influential data points were present. Note that the use of a second, independent data set in calculating the predictive capabilities of the response surface approximations (Section 2.4) also protects against possible influential data points and was the only diagnostic considered to ensure data points of good quality throughout the rest of this dissertation.

2.6. Optimization

One of the most attractive features of using response surface approximations in optimization is that due to the least squares procedure and the use of polynomial functions as response surface approximations, the resulting approximate response function is a smooth function. Having a smooth response function is important since most optimization algorithms have problems dealing with non-smooth response functions because the optimization algorithm easily converges on a local minimum. In the case of numerical experiments, the non-smooth functional behavior is a result of numerical noise, which creates spurious local minima that easily trap the optimization algorithm and make it difficult to find the best designs (e.g., Giunta³ et al. 1994).

When replacing the noisy response function obtained from the numerical experiments with a smooth response surface approximation, one effectively filters out the numerical noise,

thus allowing the use of derivative-based optimization algorithms. Derivative-based optimization algorithms are efficient algorithms and are readily available commercially. In the present dissertation, both the generalized reduced gradient algorithm provided with Microsoft Excel Version 8.0⁶⁹ (1997) and the sequential linear programming (SLP) procedure provided with the Design Optimization Tools⁷⁰ (DOT) (1995) software, were used to solve the optimization problems. Microsoft Excel Version 8.0⁶⁹ (1997) provided an easy to use, interactive environment. However, since each optimization problem is solved interactively, this environment is not suitable for performing a large number of optimizations. In contrast, the Design Optimization Tools⁷⁰ (1995) software is supplied as a number of FORTRAN⁷¹ (1995) subroutines. These subroutines can easily be called from a simple FORTRAN⁷¹ (1995) program to perform a large number of optimizations with no further interaction required from the designer.

CHAPTER 3

FUZZY SET THEORY

Fuzzy set theory was introduced by Zadeh⁵³ (1965) as a mathematical tool for the quantitative modeling of uncertainty, and makes use of fuzzy numbers to represent uncertain problem parameters. In order to represent an uncertain parameter as a fuzzy number, the designer only needs to specify the range of uncertainty and a membership function that denotes the possibility of occurrence of an element in the specified range. Membership functions are generally constructed subjectively, based on expert opinion. Fuzzy set theory caters to worst case scenarios and thus compensates for the fact that the uncertainty is modeled based on subjective opinions and assumptions (Maglaras et al.⁵² 1997).

3.1. Fuzzy Sets, Fuzzy Numbers and Fuzzy Functions

Fuzzy set theory presents a methodology for the mathematical modeling of uncertainty. In contrast to classical (or crisp) set theory where an element either completely belongs to a specific set or not at all, fuzzy set theory makes use of membership functions to denote the degree to which an element belongs to a fuzzy set.

Any crisp set has a characteristic equation that assigns a value of either 0 or 1 to each member of the universal set, where 0 is generally used to denote non-membership and 1 to denote membership. An example is a set B defined such that x is any real number between 2 and 3, which may be written as:

$$B = \{x | 2 \leq x \leq 3, x \in \mathfrak{R}\} \quad (3.1)$$

The characteristic equation for set B is then defined as:

$$\psi_B = \begin{cases} 1 & \text{for } x \in B \\ 0 & \text{for } x \notin B \end{cases} \quad (3.2)$$

Thus, for the crisp set B the characteristic equation maps elements of the set $\mathfrak{R}$ to elements of the set $\{0,1\}$ as follows:

$$\psi_B : \mathfrak{R} \rightarrow \{0,1\} \quad (3.3)$$

A fuzzy set is defined by generalizing the characteristic equation to assign a grade of membership, ranging between 0 and 1, to each element of the universal set. In fuzzy set theory, this generalized characteristic equation is referred to as a membership function. The notation used to denote a membership function is as follows:

$$\mathbf{M}(x):X \rightarrow [0,1] \quad (3.4)$$

Where $\mathbf{M}$ denotes the membership function, mapping the elements of the universal set X to the real interval $[0, 1]$. In the present dissertation the same symbol, a bold face capital letter, is used to denote both the fuzzy set and its membership function. Since each fuzzy set is completely and uniquely defined by only one particular membership function, no ambiguity results from the double use of the same symbol.

Fuzzy sets may be represented numerically by making use of α level cuts. An α level cut is defined as the real interval where the membership function is larger than a given value α (Klir and Yuan⁷² 1995, p. 19) and may be written mathematically for a generic fuzzy set $\mathbf{B}$ as follows:

$${}^{\alpha}B = \{x | \mathbf{B}(x) \geq \alpha\} \quad (3.5)$$

Equation (3.5) is shown graphically in Fig. (3.1), where it is assumed that $\mathbf{B}$ has a triangular and symmetric membership function. Also shown in Fig. (3.1) are the lower and upper bounds of the α level cut, ${}^{\alpha}b_1$ and ${}^{\alpha}b_2$.

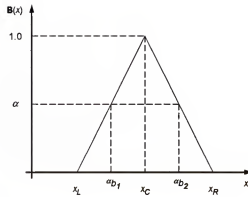


Figure 3.1: Triangular and symmetric membership function with α level cut shown.

A fuzzy number is defined as a fuzzy set that is both normal and convex (Klir and Yuan⁷² 1995, p. 97). A normal fuzzy set is one for which the maximum membership function is equal to 1, while a convex fuzzy set is one for which all possible α level cuts are convex. The fuzzy set **B** shown in Fig. (3.1) is thus a fuzzy number. In fact, the triangular and symmetric membership function is most often used to represent fuzzy numbers, mainly due to its simplicity. As shown in Fig. (3.1) any triangular fuzzy number (not necessarily symmetric) may be represented by only three variables: x_L ; x_C ; and x_R .

A fuzzy function **Y** is a function of fuzzy variables X_i and may be written as

$$Y = Y(X_1, X_2, \dots, X_n) \quad (3.6)$$

for the case where n fuzzy variables are considered. The following properties of fuzzy functions are summarized and proved by Klir and Yuan⁷² (1995, pp. 105-109):

1. When all of the fuzzy variables of a fuzzy function are continuous fuzzy numbers, the fuzzy function itself is also a continuous fuzzy number.

2. When all of the fuzzy variables of a fuzzy function are fuzzy numbers, the α level cut of a fuzzy function ${}^{\alpha}Y$ may be written in terms of the α level cuts of its fuzzy variables ${}^{\alpha}X_i$ as follows

$$\begin{aligned} {}^{\alpha}Y &= Y({}^{\alpha}X_1, {}^{\alpha}X_2, \dots, {}^{\alpha}X_n) \\ &= \left[\min_{{}^{\alpha}R} \left(Y({}^{\alpha}X_1, {}^{\alpha}X_2, \dots, {}^{\alpha}X_n) \right), \max_{{}^{\alpha}R} \left(Y({}^{\alpha}X_1, {}^{\alpha}X_2, \dots, {}^{\alpha}X_n) \right) \right] \end{aligned} \quad (3.7)$$

where ${}^{\alpha}R$ denotes the n -dimensional rectangle, formed by the α level cuts of the n fuzzy numbers.

3.2. The Vertex Method

Based on the properties of a fuzzy function (see Eq. 3.7), Dong and Shah⁶⁶ (1987) introduced the vertex method for evaluating the upper and lower bounds of ${}^{\alpha}Y$ when all the fuzzy variables of Y are fuzzy numbers. When ${}^{\alpha}Y$ has no global extreme points in the interior (including the boundaries) of the n -dimensional rectangle formed by the α level cuts of the n fuzzy numbers, the vertex method yields

$${}^{\alpha}Y = \left[\min_j \left(Y({}^{\alpha}c_j) \right), \max_j \left(Y({}^{\alpha}c_j) \right) \right] \quad \forall j = 1, 2^n \quad (3.8)$$

where ${}^{\alpha}c_j = ({}^{\alpha}X_1, {}^{\alpha}X_2, \dots, {}^{\alpha}X_n)_j$ denotes the coordinates of the 2^n vertices of the n -dimensional rectangle obtained from the fuzzy numbers for a specific α value. When the fuzzy function has global extreme points in this n -dimensional rectangle (including the boundaries), the function evaluations at these extreme points must be included when evaluating the minimum and maximum operators of Eq. (3.8). Note that since all possible combinations of the α level cut end points of the fuzzy numbers are considered, the evaluation of Eq. (3.8) generally requires a large number of function evaluations and is computationally intensive.

In order to ensure that no global extreme points exist within the design space (including the boundaries), except at the vertices, the sequential linear programming procedure provided with the Design Optimization Tools (DOT)⁷⁰ (1995) software was used. A large number of derivative-based optimizations from different starting points, randomly distributed throughout the design space, were performed for a small number of representative α level cut values. The results obtained from these optimizations were then investigated to ensure that no global extreme points existed at locations other than the vertices of the design space.

3.3. Possibility Measure and Possibility Distribution

A fuzzy number may also be considered as the trace of a possibility measure Π on the singletons (single elements) x of the universal set X (Dubois and Prade⁷³ 1988, pp. 13-17). When a possibility measure defined on the unit interval is considered, its possibility distribution π can then be interpreted as the membership function of a fuzzy number **B** describing the event that Π focuses on, which may be written mathematically as:

$$\Pi(\{x\}) = \pi(x) = \mathbf{B}(x) \quad \forall x \in X \quad (3.9)$$

The possibility of an event is calculated from the possibility distribution function and is related to the maximum value of the possibility distribution function. The calculation of the possibility of an event may be illustrated with a simple example problem where a real number x that is approximately equal to 2 is considered. This number may be represented by a fuzzy set **B** with a symmetric and triangular membership function (see Fig. 3.2) where it is assumed that real numbers between 1.5 and 2.5 have non-zero possibility of being approximately equal to 2.

Recall that according to Eq. (3.9) the membership function of Fig. (3.2) is equivalent to the possibility distribution for the event that a given real number x is approximately equal

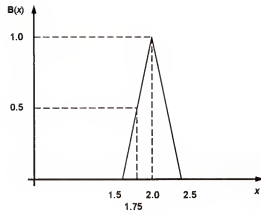


Figure 3.2: Possibility distribution for x approximately equal to 2.

to 2. The possibility that the value of x would fall in a specified range, say from 1.5 to 1.75, is equal to the maximum value of the possibility distribution function (i.e., the membership function) over that range, in this case 0.5. Similarly, the possibility that the value of x would fall in the range from 1.5 to 2.0 is equal 1.0, which is the same possibility that the value of x would fall in the range from 1.75 to 2.0. Additionally, the possibility that the value of x is exactly equal to a specific value, say 1.75, is equal to the value of the possibility distribution at that value, in this case equal to 0.5. Similarly, the possibility that the value of x is equal to 2.0 would thus be equal to 1.0.

Calculating the possibility of an event is significantly different from calculating the probability of the same event. The probability of an event is calculated from a probability distribution function and is related to the area under the probability distribution function. To illustrate calculating the probability of an event, an uniform probability distribution function may be used to represent the real number x that is approximately equal to 2. The uniform probability distribution function indicates that the value of x has an equal probability of being equal to any real number in the range 1.5 to 2.5, and is shown in Fig. (3.3).

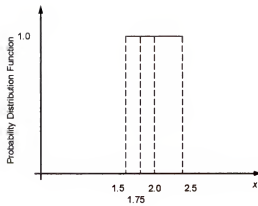


Figure 3.3: Uniform probability density function for x approximately equal to 2.

Note that a probability distribution function is always constructed so that the total area under the graph is equal to 1. The probability that the value of x would fall in a specified range, say from 1.5 to 1.75, would then be equal to the area under that part of the probability distribution function, in this case 0.25. Similarly, the probability that the value of x would fall in the range from 1.75 to 2.0 is also equal to 0.25 and in the range from 1.5 to 2.0 is equal to 0.5. Additionally, the probability that the value of x would be equal to a specific number, for example 1.75 or 2.0, is equal to zero since the area under that part of the graph (a point) is equal to zero.

The difference between the probability and possibility of an event stems from the fact that probability and possibility are two different measures of uncertainty (Dubois and Prade⁷³ 1988, pp. 1-4). Probability theory is concerned with the frequency of the occurrence of an event. For our example problem, the event may be drawing a card from a bag with an infinite number of cards each denoting a real number between 1.5 and 2.5. Thus, the probability of drawing a card that has a value exactly equal to a specific real number (say 1.75) would be equal to 0, while drawing a card with a number in the range 1.5 to 1.75 would be equal to 0.25.

In contrast, the possibility of an event is a measure of how plausible that event is, in other words how surprised you would be if you knew x is approximately equal to 2.0 and the actual value turns out to be equal to 1.75 for example. A more detailed discussion on the differences between probability theory and possibility theory are provided in Dubois and Prade⁷³ (1988, pp. 1-13) and Klir and Yuan⁷² (1995, pp. 200-208).

Finally, for calculating the possibility of failure it is often required to compare a crisp number with a fuzzy number. The possibility measure of a crisp number x being smaller or equal to a fuzzy number $\mathbf{B}$ is defined by Dubois and Prade⁷³ (1988, pp. 99-101) as follows:

$$\Pi_{\mathbf{B}}([x, +\infty)) = \sup_{y \geq x} \mathbf{B}(y) \quad \forall x \quad (3.10)$$

The possibility distribution function $\pi_{\mathbf{B}}$ corresponding to the possibility measure of Eq. (3.10) is shown graphically in Fig. (3.4) for the general case where $\mathbf{B}$ has a non-linear membership function.

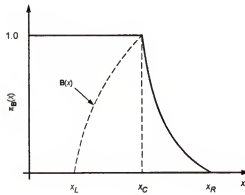


Figure 3.4: Possibility distribution for $\mathbf{B} \geq x$ with non-linear $\mathbf{B}(x)$.

Using the definition of Eq. (3.10), the possibility distribution of failure $\pi_{(\mathbf{P}-\mathbf{P}_f)}$ is obtained from the fuzzy function $(\mathbf{P} - \mathbf{P}_f)$ which contains the fuzzy numbers $\mathbf{P}$ (the applied load) and $\mathbf{P}_f$ (the failure load of the structure) as fuzzy variables. The possibility of failure is then defined as:

$$\Pi_{(\mathbf{P}-\mathbf{P}_f)}([0,+\infty)) = \sup_{y \geq 0} (\mathbf{P} - \mathbf{P}_f)(y) \quad (3.11)$$

In the present dissertation, the programming languages FORTRAN⁷¹ (1995) and JAVA⁷⁴ (1996) were used to develop software for calculating the possibility of failure where all uncertain input parameters were represented by triangular and symmetric membership functions.

CHAPTER 4 ISOTROPIC PLATE RESPONSE SURFACE CONSTRUCTION

Designing for uncertainty of an isotropic plate with a change in thickness across its width is the first example problem that will be considered. The uncertainty considered is typical of the aircraft industry where structures are often designed that will be built well into the future from materials not yet available. Oftentimes these materials are not yet available and the designer is forced to deal with uncertain material properties. In addition, the cost of manufacturing is also uncertain. However, unlike the material properties, the designer has some control over the manufacturing cost which is closely linked to the required tolerances in geometry. For such design problems, little information about the uncertainty is known and the uncertainty is typically based on expert opinion and assumptions made by the designer. Fuzzy set theory is ideally suited for this class of problems and is capable of modeling uncertainty using limited data available to the designer.

4.1. Problem Description

An isotropic plate with a change in thickness in the form of a linear ramp (see Fig. 4.1) is the first design problem considered in the present dissertation. The plate is simply supported on two of its edges, free on the other two edges, and subjected to a uniformly distributed load, applied on the two simply supported edges. Three non-dimensional parameters λ , β and γ specify the geometry and location of the change in thickness (see Fig. 4.2). These variables vary in the following intervals: $\lambda \in [0,1]$, $\beta \in [-0.5,0.5]$ and $\gamma \in [0,1]$.

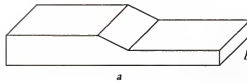


Figure 4.1: Three dimensional perspective.

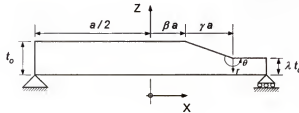


Figure 4.2: Cross-section model with non-dimensional variables shown.

Yielding is considered as a failure mechanism in the design, while the plate is also designed to be buckling resistant. The Von Mises failure criterion is used to calculate the failure load of the plate due to yielding. For the buckling load constraint, the design fails if the applied load exceeds the buckling load. Response surface approximations for the stress distribution near the re-entrant corner and for the buckling load were constructed in terms of the geometry and location of the change in thickness. Numerical experiments were conducted using finite element models of the plate with MSC/NASTRAN Version 68⁷⁵ (1994). A cross-section of the plate was used to model the stress distribution near the re-entrant corner, using four-node, isoparametric, plane strain elements. All models had a uniform finite element mesh, typically having 1,800 elements in the x -direction and 9 elements in the z -direction. The number of elements varied slightly from model to model. A schematic representation of the finite element model used is shown in Fig. (4.3).

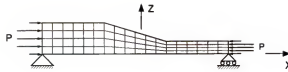


Figure 4.3: Schematic finite element model used to determine the stress distribution near the re-entrant corner (actual model has about 16,000 elements).

For the buckling response, constant thickness, four-node, isoparametric plate bending elements were used to construct a two-dimensional twenty element by twenty element finite element model similar in shape to a plan view of Fig. (4.1), as shown in Fig (4.4). The thickness of the elements in the ramp region is the average thickness over the element. The eccentricity of the mid-plane was found to have an insignificant effect on the Euler buckling load value and was ignored in the analysis. The eccentricity is expected to cause large deflections when the applied load approaches the buckling load value, which usually results in stress failure.

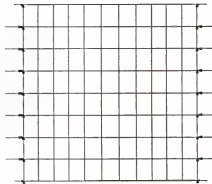


Figure 4.4: Schematic finite element model used to determine the buckling load (actual model has 400 elements).

4.2. Dimensional Analysis

By reducing the number of variables in a response surface approximation, the accuracy and the usefulness of the response surface approximation are increased. The number of

variables in the response surface approximations of the present example was reduced by using variables that are intrinsic to the problem, identified by performing a dimensional analysis of the governing equations.

4.2.1. Stress Distribution near the Re-entrant Corner of the Plate

The theoretical value of the stress at the re-entrant corner is infinite. However, due to yielding and the impossibility of manufacturing an absolutely sharp corner, the actual stress at the corner will be finite. To account for yielding, the Von Mises criterion is used to define failure in terms of a maximum allowable yield zone near the re-entrant corner, with failure occurring when the plate yields outside of this zone. For plane strain, the radial stress at a distance r from the re-entrant corner with angle θ (see Fig. 4.2), is (Williams⁷⁶ 1952)

$$\sigma \sim r^{\zeta-1} \quad (4.1)$$

where ζ is solved from the following transcendental equation:

$$\sin(\zeta\theta) = \pm\zeta\sin(\theta) \quad (4.2)$$

To avoid the need for repetitively solving Eq. (4.2), a third order polynomial approximation of ζ as a function of θ (in radians) was constructed. Nineteen ζ values corresponding to equally spaced values of θ between π and $3\pi/2$ were used to obtain:

$$\zeta = 7.457 - 4.046\theta + 0.8061\theta^2 - 0.05491\theta^3 \quad (4.3)$$

The %MaxErr value (see Eq. 2.21) for the data points used to construct the approximation is 0.41%.

The plane strain governing equations, obtained from the equilibrium equations, the stress-strain and strain-displacement relationships (Cook, Malkus and Plesha⁷⁷ 1989, pp. 20-21), depend on nine parameters: λ , β , γ , a , b , t_∞ , P , E and ν . By defining the non-dimensional parameters

$$\begin{aligned}
\tilde{x} &= x/a & \tilde{z} &= z/t_o \\
\tilde{u} &= (Et_o/P)u & \tilde{v} &= (Ea/P)v \\
\tilde{\epsilon}_x &= (Eat_o/P)\epsilon_x & \tilde{\sigma}_x &= (\lambda bt_o/P)\sigma_x \\
\tilde{\epsilon}_z &= (Eat_o/P)\epsilon_z & \tilde{\sigma}_z &= (\lambda bt_o/P)\sigma_z \\
\tilde{\gamma}_{xz} &= (Eat_o/P)\gamma_{xz} & \tilde{\tau}_{xz} &= (\lambda bt_o/P)\tau_{xz}
\end{aligned} \tag{4.4}$$

the governing equations may be written in non-dimensional form as:

$$\begin{aligned}
2(1-\nu)(t_o/a)\tilde{u}_{,\tilde{z}\tilde{z}} + (t_o/a)\tilde{v}_{,\tilde{z}\tilde{z}} + (1-2\nu)(a/t_o)\tilde{u}_{,\tilde{x}\tilde{x}} &= 0 \\
(a/t_o)\tilde{u}_{,\tilde{x}\tilde{x}} + (1-2\nu)(t_o/a)\tilde{v}_{,\tilde{z}\tilde{z}} + 2(1-\nu)(a/t_o)\tilde{v}_{,\tilde{x}\tilde{x}} &= 0
\end{aligned} \tag{4.5}$$

The associated non-dimensional boundary conditions are summarized in Table (4.1).

Table 4.1: Non-dimensional boundary conditions.

Boundary	Prescribed Stress
$\tilde{x} = -1/2$	$\tilde{\sigma}_x = -\lambda \quad \tilde{\tau}_{xz} = 0$
$\tilde{x} = 1/2$	$\tilde{\sigma}_x = 1 \quad \tilde{\tau}_{xz} = 0$
$\tilde{z} = 0$	$\tilde{\sigma}_z = 0 \quad \tilde{\tau}_{xz} = 0$
$\tilde{z} = 1$	$\tilde{\sigma}_z = 0 \quad \tilde{\tau}_{xz} = 0$
$\tilde{z} = \lambda$	$\tilde{\sigma}_z = 0 \quad \tilde{\tau}_x = 0$
$(1/2 + \beta) \leq \tilde{x} \leq (1/2 + \beta + \gamma)$ and $\tilde{z} = m\tilde{x} + c$ $m = -(1-\lambda)/\gamma$ $c = \frac{1-\lambda+2\beta-2\lambda\beta+2\gamma}{2\gamma}$	
<u>Normal Stress:</u> $\tilde{\sigma}_N = 0$ <u>Surface Shear:</u> $\tilde{\tau}_s = 0$	

From Eq. (4.5) and Table (4.1), λ , β , γ , a/t_o and ν are identified as the intrinsic variables associated with the plane strain solution. However, the stress distribution near the re-entrant corner also depends on the radial distance measured from the corner (see Eq. 4.1). An investigation of several plate configurations revealed that the maximum stress always occurs

on the top surface of the thinner section of the plate where σ_x is the only non-zero stress component, and the Von Mises failure criterion reduces to

$$\tilde{\sigma}_x \geq \tilde{\sigma}_r = \frac{\lambda b t_o \sigma_r}{P} \quad (4.6)$$

where σ_r denotes the material yield stress. By defining the non-dimensional distance

$$\tilde{r} = \frac{r}{t_o} \quad (4.7)$$

the response surface approximation of the non-dimensional stress distribution on the top surface of the thinner section of the plate may be approximated as

$$\tilde{\sigma}_x = C_1(\lambda, \beta, \gamma) + C_2(\lambda, \beta, \gamma) \tilde{r}^{\zeta-1} \quad (4.8)$$

where ζ also depends on λ , γ and a/t_o through the angle θ . In the present dissertation, response surface approximations were constructed to represent the unknown functions C_1 and C_2 of Eq. (4.8).

Thus, there are seven intrinsic variables (λ , β , γ , a/t_o , ν , $\tilde{r}$ and ζ) associated with the solution for the stress distribution near the re-entrant corner. However, it is expected that the non-dimensional stress distribution varies only for small values of a/t_o (thick plates), and that for large a/t_o values (thin plates), the influence of a/t_o may be ignored. Additionally, it is also anticipated that small changes in Poisson's ratio about the baseline value of 0.3 should not have a large influence on the non-dimensional stress distribution. For the case where $\lambda=0.5$, $\beta=0$, $\gamma=0$ (corresponding to $\theta=3\pi/2$ and $\zeta=0.55$) and $\tilde{r}=0.1$ (10% of t_o), varying a/t_o between 1 and 400 changes the non-dimensional stress by less than 1% for a/t_o larger than 25. Furthermore, for the case where $\lambda=0.5$, $\beta=0$, $\gamma=0$, $\tilde{r}=0.03$ and $\zeta=0.55$, varying the value of ν from 0.2 to 0.4 changes the non-dimensional stress by only 1.85%. In addition to these

evaluations for a single design point, random combinations of λ , β and γ were tested as discussed subsequently in Section 4.3.1.1.

Hence, when $\nu \approx 0.3$ and $a/t_o \geq 25$, the $\tilde{\sigma}_x$ stress on the top surface of the thin section of the plate may be described in terms of only four predictor variables λ , β , γ and $\tilde{r}^{\zeta^{-1}}$, where $\tilde{r}$ and ζ are combined into a single variable $\tilde{r}^{\zeta^{-1}}$. Recall that predictor variables are the variables of the response surface approximation used to approximate $\tilde{\sigma}_x$.

4.2.2. Buckling Load of the Plate

The buckling load of the plate depends on eight parameters: λ , β , γ , a , b , t_o , E and ν . The flexural stiffness of the plate is given by

$$D = \frac{Et^3}{12(1-\nu^2)} \quad (4.9)$$

where the thickness t is a function of t_o , λ , β , r and x . The non-dimensional parameters

$$\begin{aligned} \tilde{x} &= \frac{x}{a} & \tilde{y} &= \frac{y}{a} \\ \tilde{t} &= \frac{t}{a} & \tilde{w} &= \frac{w}{a} \\ \tilde{N}_x &= \frac{N_x}{P_{Euler}} & \tilde{D} &= \frac{D}{Ea^3} = \frac{\tilde{t}^3}{12(1-\nu^2)} \end{aligned} \quad (4.10)$$

are defined, and

$$P_{Euler} = \frac{\pi^2 \bar{D}}{a^2} \quad (4.11)$$

where $\bar{D}$ is a representative value of D , defined subsequently in Eq. (4.19). Then the governing equation and boundary conditions for the buckling load due to uniform axial compression (Timoshenko⁷⁸ 1936, p. 297 and pp. 337-350) may be non-dimensionalized to obtain

$$\begin{aligned}
& (\tilde{D}\tilde{w}_{,xx})_{,xx} + 2(\tilde{D}\tilde{w}_{,xy})_{,xy} + (\tilde{D}\tilde{w}_{,yy})_{,yy} - \nu(\tilde{D}\tilde{w}_{,yy})_{,xx} - \nu(\tilde{D}\tilde{w}_{,xx})_{,yy} \\
& + 2\nu(\tilde{D}\tilde{w}_{,xy})_{,xy} = -\pi^2 \tilde{D}\tilde{N}_s \tilde{w}_{,xx}
\end{aligned} \quad (4.12)$$

where:

$$\begin{aligned}
& \text{For } \tilde{x} = -a/2 \text{ and } \tilde{x} = a/2: & \tilde{w} &= 0 & \tilde{w}_{,xx} &= 0 \\
& \text{For } \tilde{y} = -\frac{b}{2a} \text{ and } \tilde{y} = \frac{b}{2a}: & \tilde{w}_{,yy} + \nu\tilde{w}_{,xx} &= 0 & \tilde{w}_{,yyy} + (2-\nu)\tilde{w}_{,xyy} &= 0
\end{aligned} \quad (4.13)$$

The value of $\tilde{N}_c$ corresponding to the buckling load is denoted by $\tilde{N}_{crit}$. Five intrinsic variables associated with the solution for the buckling load are identified as: ν , a/b , λ , β and γ . The non-dimensionalization of Eq. (4.10) is based on Euler buckling of the plate. The non-dimensional buckling load is independent of Poisson's ratio and a/b because the Euler buckling load is independent of these parameters. However, for large values of a/b the structure behaves like a beam, rather than a plate, with the buckling load being higher by a factor of $1/(1-\nu^2)$. The dependence of the non-dimensional buckling load on the variables ν and a/b was tested by varying these variables for the case $\lambda=0.5$, $\beta=0$ and $\gamma=0$. It was found that $\tilde{N}_{crit}$ changes by 3.90% when ν is varied from 0.2 to 0.4 ($a/b=1$), while $\tilde{N}_{crit}$ changes by 7.5% when a/b is varied from 0.5 and 10 ($\nu=0.3$). Note that the relatively large error of 7.5% is due to the fact that the structure behaves like a beam when $a/b=10$. Additional evaluations of the magnitude of the dependence of the buckling load on Poisson's ratio and a/b at other random design points are described subsequently in Section 4.3.1.2.

Hence, for a plate with a Poisson's ratio value close to 0.3, the response surface approximation for $\tilde{N}_{crit}$ has only three predictor variables λ , β and γ .

4.3. Response Surface Approximation Construction

The objective of the current design problem is to maximize a safety measure of the plate with change in thickness for a given weight. In the present dissertation, the factor of safety is the safety measure considered for deterministic design, while one minus the possibility of failure is the safety measure considered for fuzzy set based designing for uncertainty. When considering both the yielding (Von Mises yield stress failure criterion) failure mechanism and the buckling response constraint, the failure load P_f of the plate is given by

$$P_f = \text{Min} \begin{cases} \text{Yield Stress} \\ \text{Buckling Load} \end{cases} \quad (4.14)$$

with failure defined to occur when:

$$P - P_f \geq 0 \quad (4.15)$$

In the present example problem, response surface approximations form an integral part of the fuzzy set based design. As outlined in Fig. (4.5), two levels of response surface approximations are employed during different stages of the design process. On the first level,

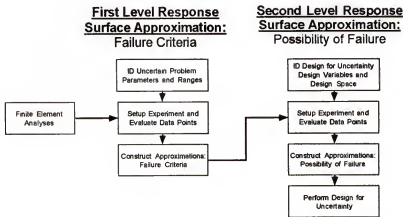


Figure 4.5: Summary of two level response surface approximation construction.

response surface approximations of the non-dimensional stress distribution $\tilde{\sigma}_x$ and non-dimensional buckling load $\tilde{N}_{crit}$ are constructed and are used to replace computationally expensive finite element analyses during the evaluation of the possibility of failure of the structure. These response surface approximations are constructed based on finite element analyses and are used mainly to reduce the computational burden associated with calculating the possibility of failure using the vertex method.

On the second level, response surface approximations of the possibility of failure as a function of the nominal values of the design variables and the level of uncertainty associated with these variables are constructed. These approximations are constructed based on results obtained from the vertex method, using the first level response surface approximations. The second level response surface approximations are constructed mainly to simplify the integration of the analysis code with the optimization algorithm, as well as to filter out numerical noise by providing a smooth approximate response function. Filtering out noise in the approximate response function allows the use of a derivative-based optimization algorithm. The generalized reduced gradient algorithm provided with Microsoft Excel Version 8.0⁶⁹ (1997) was used in the present example problem.

4.3.1. Stress Distribution and Buckling Load Response Surface Approximations

From Section 4.2.1, the stress distribution response surface approximation may be written in functional form as follows:

$$\tilde{\sigma}_x = \tilde{\sigma}_x(\lambda, \beta, \gamma, \tilde{r}^{\zeta-1}) \quad (4.16)$$

The behavior of the stress distribution near the re-entrant corner of the plate was determined by varying each of the four predictor variables individually for specific combinations of the other three variables. It was determined that a fourth order polynomial

in the four predictor variables (70 parameters) was suitable for an initial response surface approximation.

The buckling load response was similarly investigated and the resulting graphs indicated that the behavior of the buckling load response is more complex than the stress distribution response. The additional complexity is mainly due to the presence of two buckling modes. A local buckling mode is active for high slenderness ratios of the thin section of the plate, while a global buckling mode is active otherwise. To model these two buckling modes accurately, it was necessary to construct two independent response surface approximations, one for each buckling mode. These response surface approximations may be written in functional form (from Section 4.2.2) as:

$$\tilde{N}_{Local} = \tilde{N}_{Local}(\lambda, \beta, \gamma) \quad \tilde{N}_{Global} = \tilde{N}_{Global}(\lambda, \beta, \gamma) \quad (4.17)$$

The necessity of two separate response surface approximations implies that the buckling load data, obtained from the finite element analyses, must be divided into two groups corresponding to the two buckling modes. A simple geometric criterion, which assigns data points to the appropriate buckling mode response surface approximation, was defined based on the slenderness ratio of the thin section of the plate. This criterion is written in non-dimensional form as:

$$\text{Buckling mode} = \begin{cases} \text{Local mode if } \frac{(0.5 - \beta - \gamma)}{\lambda} \geq 0.6 \\ \text{Global mode if } \frac{(0.5 - \beta - \gamma)}{\lambda} < 0.6 \end{cases} \quad (4.18)$$

Two separate definitions of the $\bar{D}$ value of the Euler buckling load used in the non-dimensionalization of Eq. (4.10) was defined based on the local and global buckling modes respectively. For the local buckling mode, the thickness of the thin section of the plate, λ_s ,

was used as an equivalent thickness value, while the average t^3 value was used as an equivalent thickness value for the global buckling mode. The $\bar{D}$ value is then defined as:

$$\bar{D} = \begin{cases} \frac{E(\lambda t_o)^3}{12(1-\nu^2)} & \text{if local buckling mode} \\ \frac{E(\bar{t}^3)_{avg}}{12(1-\nu^2)} & \text{if global buckling mode} \end{cases} \quad (4.19)$$

After investigating several different order polynomials, it was determined that a third order polynomial (20 parameters) is suitable as a response surface approximation for the local buckling mode, and a fourth order polynomial (35 parameters) is suitable as a response surface approximation for the global buckling mode.

The design space used for constructing the stress distribution and buckling load response surface approximations is summarized in Table (4.2) and is shown graphically for λ , β and γ in Fig (4.6). In Table (4.2) the upper limit on $\tilde{r}$ limits the radius of the yield zone

Table 4.2: Design space for constructing the response surface approximations.

Response Variable	Range
λ	$0.2 \leq \lambda \leq 1.0$
β	$-0.475 \leq \beta \leq 0.475$
γ	$0 \leq \gamma \leq 0.475 - \beta$
$\tilde{r}^{c-1}$	$0 \leq \tilde{r} \leq 0.8\lambda$

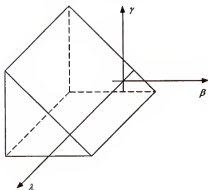


Figure 4.6: Three dimensional representation of the design space.

about the re-entrant corner to be no greater than 80% of the thickness of the thin section of the plate. The upper bound on γ is dictated by the geometry of the transition region.

For the isotropic plate example problem, each finite element analysis was computationally inexpensive to evaluate. Since it was feasible to perform a large number of numerical experiments, it was not necessary to use statistical design of experiments for selecting a small number of data points at which to evaluate the response function. Instead, the response function was evaluated at a large number of data points. Although this approach is rarely practical, it provided a large number of data points that were used as an independent data set for evaluating the predictive capabilities of the response surface approximations. This independent data set was used to validate the proposed methodology before considering the second, more complicated, example problem. The numerical experiments were conducted for a three-dimensional grid of equally spaced data points with eight data points in each of the λ , β and γ directions. For a rectangular domain, this procedure would yield a total of 512 data points. However, due to the prism or wedge-like domain of Fig. (4.6), only 288 of these 512 data points fell within the current design space.

4.3.1.1. Stress distribution response surface approximation

For the stress distribution, each of the 288 plate configurations (corresponding to 288 finite element analyses) included a number of data points with different $\tilde{r}^{z-1}$ values (corresponding to different finite elements). The stress values for all finite elements on the top surface of the thin section of the plate were recorded for node points within 80% of the thickness of the thin section of the plate, measured from the re-entrant corner. This process yielded a total of 3,023 data points that were divided into two groups by assigning a random number between 0 and 1 to each data point. Data points with a random number less than 0.7

were allocated to the set of data points used to construct the response surface approximation, yielding 2,124 data points (Data Set 1). The remaining 899 data points were reserved to evaluate the predictive capabilities of the response surface approximation (Data Set 2).

Using the mixed stepwise regression procedure, the optimum number of parameters in a reduced response surface approximation for the stress distribution was found to be 43. Using both Data Set 1 and Data Set 2, the predictive capabilities of the initial response surface approximation (70 parameters) and of the reduced response surface approximation (43 parameters) can be compared by comparing the results shown in Table (4.3).

Table 4.3: Stress distribution response surface approximation.

Model	C _p	R ²	Adj-R ²	%RMSE	%RMSE PRESS	%Avg Err	%RMSE	%Max Err
Stress	Data Set 1 (2,124 Data Points)				Data Set 2 (899 Data Points)			
Full (70 terms)	70	0.9984	0.9983	3.2478	3.4172	2.4552	3.5934	16.443
Reduced (43 terms)	31.927	0.9983	0.9982	3.2964	3.3886	2.4339	3.4754	17.120

The results in Table (4.3) indicate that both models have similar predictive capabilities. The reduced response surface approximation is slightly more accurate (except for the %MaxErr values), and is preferred since it represents a smoother function due to a smaller number of parameters. The error terms obtained from Data Set 1 are slightly optimistic due to the fact that they are calculated from data points used to construct the response surface approximation. However, the results obtained from the %RMSE_{PRESS} value compare well with the results obtained from Data Set 2. This trend is a general trend when the number of data points is much larger than the number of parameters in the response surface approximation.

Figure (4.7) shows the correlation between the non-dimensional finite element stress values and the non-dimensional stress values obtained from the response surface approximation. The correlation for the data points of Data Sets 1 and 2 is shown as circular symbols. If there were 100% correlation between the finite element and the response surface approximation stress values, the circular symbols would lay on the ideal diagonal line. From Fig. (4.7) it is clear that the circular symbols are close to the ideal diagonal line, indicating a high correlation or accuracy between the finite element and response surface approximation stress values.

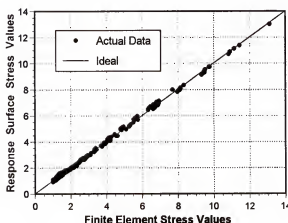


Figure 4.7: Response surface approximation compared to finite element non-dimensional stress values (Data Sets 1 and 2).

The ability of the response surface approximation to predict accurate values for Poisson's ratios that are different from 0.3 was evaluated with ten random combinations of λ , β and γ . The values $\nu=0.25$ and $\nu=0.35$ were used for each configuration. The average error for this evaluation set was 3.3%, indicating that small changes in Poisson's ratio do no effect the results.

An important and desirable characteristic of response surface approximations is their tendency to filter out numerical noise. This characteristic is illustrated for the maximum

stress value on the top surface of the thin section of the plate (independent of $\tilde{r}$) as a function of λ for $\beta=\gamma=0$. These values of β and γ correspond to a sharp transition at the mid-length of the plate. For this illustration, 60 data points were evaluated and the experimental results are shown as circular symbols in Fig. (4.8). The response obtained from the response surface approximation is shown as a solid line in Fig. (4.8).

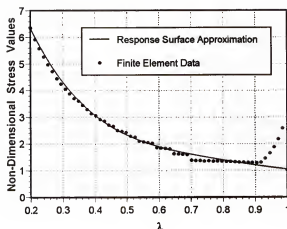


Figure 4.8: Maximum non-dimensional stress value as a function of λ ($\beta=0$, $\gamma=0$).

There are some inconsistencies in the data obtained from the finite element model, which may be attributed to the mesh generator that did not provide finite element models with a consistent level of fidelity. In particular, the mesh generator yielded less accurate finite element models when γ is equal to 0 and λ is close to 1, corresponding to a sharp transition in thickness with a small change in thickness. The results presented in Fig. (4.8) indicate that the response surface approximation eliminated these inconsistencies, or noise, and indicate that not all differences between predicted and experimental response values should be considered to be errors. For some cases, the predicted response values may indeed be more accurate than the experimental response values as is the case in Fig. (4.8) when λ is close to 1. When λ is close to 1, the non-dimensional finite element stress values should approach a value equal to 1

and are thus in error. The problem with the mesh generator disappears when λ is equal to 1 (no change in thickness) since the non-dimensional finite element stress value for this design point is also equal to 1, corresponding to no stress concentration.

4.3.1.2. Buckling load response surface approximation

For representing the buckling load, Eq. (4.18) was used to divide the 288 data points into 126 data points for the construction of the local buckling mode response surface approximation, and 162 data points for the global buckling mode response surface approximation. An additional set of 196 data points was generated to be equally spaced, midway between the existing 288 data points, with 83 of these data points used for evaluating the local buckling mode response surface approximation, and 113 of these data points used for evaluating the global buckling mode response surface approximation.

Using the mixed stepwise regression procedure, the best reduced response surface approximation for the local buckling load (cubic polynomial) has 19 parameters while the reduced response surface approximation for the global buckling load (quartic polynomial) has 25 parameters. The predictive capabilities of the models are summarized in Table (4.4), and a graphical representation of the results is shown in Fig. (4.9). Figure (4.9) shows the correlation between the finite element and response surface approximation non-dimensional buckling load values for the data points of Data Sets 1 and 2. As was the case in Fig. (4.7), the high accuracy of the non-dimensional buckling load response surface approximations is illustrated by the fact that the symbols representing the actual data points are very close to the ideal diagonal line.

In addition, to determine the accuracy of the response surface approximations for different values of ν and a/b , ten random combinations of λ , β and γ were tested for $\nu=0.25$

Table 4.4: Buckling modes response surface approximations.

Model	Cp	R ²	Adj-R ²	%RMSE	%RMSE PRESS	%Avg Err	%RMSE	%Max Err
Local Buckling		Data Set 1 (126 Data Points)				Data Set 2 (83 Data Points)		
Full (20 terms)	20	0.9999	0.9998	0.5541	0.6927	0.7056	0.9567	2.8494
Reduced (19 terms)	19.402	0.9999	0.9998	0.5550	0.6920	0.7055	0.9598	2.8792
Global Buckling		Data Set 1 (162 Data Points)				Data Set 2 (113 Data Points)		
Full (35 terms)	35	0.9912	0.9889	2.5538	3.1695	1.1998	1.6895	5.4340
Reduced (25 terms)	18.122	0.9910	0.9895	2.4888	3.0202	1.1936	1.6935	5.8767

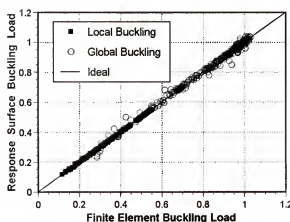


Figure 4.9: Response surface approximation compared to finite element non-dimensional buckling loads for both buckling modes (Data Sets 1 and 2).

and $\nu=0.35$, and for $a/b=0.1$ and $a/b=5$. For $a/b=0.1$ the average error was 4.5% while for $a/b=5$ the average error ranged from 2.7% for $\nu=0.25$ to 9.1% for $\nu=0.35$. This larger error reflects the fact that the structure behaves like a beam rather than a plate for $a/b=5$.

As was the case for the stress distribution response surface approximation, both the reduced and the full response surface approximations provide similar predictive capabilities. As before, the reduced response surface approximations are preferred over the full response

surface approximations, since the reduced response surface approximations represent smoother functions due to a smaller number of parameters.

4.3.2. Possibility of Failure Response Surface Approximations

The λ , β and γ design space of Table (4.2) was also used to construct the possibility of failure response surface approximation, with numerical experiments conducted on an evenly spaced grid consisting of 11 data points in each of the λ , β and γ directions. Additionally, seven evenly spaced levels of uncertainty were considered between $\pm 2\%$ and $\pm 20\%$, yielding a total of 2,629 data points in the design space. At each data point the possibility of failure values due to yielding and due to violating the buckling load constraint were evaluated from Eqs. (4.14) and (4.15) using the vertex method and the previously developed reduced stress distribution and buckling load response surface approximations.

Two response surface approximations, according to the two possibilities of failure, were constructed using all data points with possibility of failure not equal to either 0 or 1. This resulted in 499 data points for constructing the yield stress failure mode response surface approximation and 573 data points for the buckling load constraint failure response surface approximation. The resulting predicted possibility of failure is then obtained from

$$\hat{\Pi}_{(P-P_f)}(\lambda, \beta, \gamma, u) = \min(\hat{\Pi}_{YieldStress}(\lambda, \beta, \gamma, u), \hat{\Pi}_{Buckling}(\lambda, \beta, \gamma, u)) \quad (4.20)$$

where u denotes the uncertainty associated with the design variables and, as before, the caret symbol refers to predicted values.

It was found that a general fourth order polynomial (70 parameters) resulted in accurate response surface approximations for both possibility of failure response surface approximations. These general response surface approximations were reduced using the

mixed stepwise regression procedure and the C_p statistic. The predictive capabilities of the response surface approximations are summarized in Table (4.5).

Table 4.5: Possibility of failure response surface approximation.

Model	C_p	R^2	Adj- R^2	RMSE	PRESS
Stress	Fourth Order Model (499 Data Points)				
Full (70 Terms)	70.000	0.9988	0.9986	2.2525	2.6205
Reduced (59 Terms)	53.027	0.9988	0.9986	2.2371	2.5205
Buckling	Fourth Order Model (573 Data Points)				
Full (70 Terms)	70.000	0.9982	0.9980	2.7342	3.0991
Reduced (57 Terms)	48.649	0.9982	0.9980	2.7118	3.0211

For the possibility of failure response surface approximations, only a single data set was used to construct the approximations. In this case, the $\%RMSE_{PRESS}$ value was used to compensate for the possibly overly optimistic error estimates. As noted previously (see Section 4.3.1.1), the $\%RMSE_{PRESS}$ value provides good results when the number of data points used to construct the response surface approximation is much larger than the number of parameters in the response surface approximation. Since only one data set was used to construct the response surface approximations, the predictive capabilities of the response surface approximations are not presented in graphical form.

CHAPTER 5

ISOTROPIC PLATE OPTIMIZATION

The design for uncertainty problem considered here, involves maximizing a safety measure (i.e., the factor of safety for deterministic design and one minus the possibility of failure for fuzzy set based design) of an aircraft structure for a given weight. The structure considered is typical of a wing skin application where the thickness is reduced from the root to the tip, and is subject to uncertainty in the geometric variables, the material properties and the applied load. It is assumed that the structure will be built well into the future, from materials not yet available. In this case little information on the uncertainty associated with the uncertain problem parameters is available, and the uncertainty is based on expert opinion and assumptions made by the designer. Fuzzy set theory is appropriate to model uncertainty with little available information and is used in the present dissertation. Response surface approximations are used throughout the design process, mainly to reduce the computational burden associated with designing for uncertainty, but also to integrate the analysis code with the optimization algorithm. Additionally, response surface approximations filter out numerical noise in the approximate response function used during the optimization process, thus allowing the use of a derivative-based optimization algorithm.

5.1. General Design Problem

An isotropic plate with a change in thickness across its width (see Fig. 5.1) is the present design problem. All problem parameters are assumed to be uncertain. Yielding is considered as a failure mechanism, while the plate is also designed to be buckling resistant.

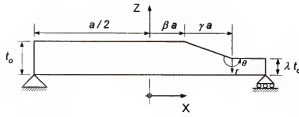


Figure 5.1: Cross-section model with non-dimensional variables shown.

The Von Mises failure criterion is used to calculate the failure load of the plate due to yielding. For the buckling load constraint, the design fails if the applied load exceeds the buckling load constraint. Based on these two criteria, the failure load P_f is calculated from

$$P_f = \text{Min} \left\{ \begin{array}{l} \frac{\sigma_y \lambda b t_o}{\tilde{\sigma}_x} \\ \frac{\tilde{N}_{Crit} \pi^2 E b (\lambda t_o)^3}{12(1-\nu^2) a^2} \end{array} \right. \quad (5.1)$$

with failure defined to occur when:

$$P - P_f \geq 0 \quad (5.2)$$

Note that throughout the design problem the buckling load N_{Crit} is non-dimensionalized with respect to the Euler buckling load of a plate with a constant thickness equal to λt_o .

The objective of the design problem is to maximize a safety measure of the structure for a given weight. The results obtained from a traditional deterministic approach, using a factor of safety to account for the uncertainty, are compared to the results obtained from a fuzzy set based approach. For the deterministic design, the factor of safety was used as the safety measure, while one minus the possibility of failure was used as the safety measure for the fuzzy set based design. In addition, the fuzzy set based analysis was also used to study the dependence of the weight of the final design on the level of uncertainty associated with the design variables λ , β and γ with the results presented in the form of a cost versus weight

design chart. Different levels of uncertainty for the design variables were considered, since these geometric variables have the largest influence on the manufacturing cost of the plate. If the tolerances of these variables can be relaxed without a large weight penalty, substantial cost reductions can be achieved in manufacturing the plate. The problem parameters and associated levels of uncertainty used in the example problem are summarized in Table (5.1).

Table 5.1: Uncertain problem parameters.

Variable	Nominal Values	Level of Uncertainty, u
$\lambda^\dagger$	[0.2 – 1.0]	$\pm 5\%$
$\beta^\dagger$	[-0.4 – 0.4]	$\pm 5\%$
$\gamma^\dagger$	[0 – 0.8]	$\pm 5\%$
a	90 in	$\pm 5\%$
b	50 in	$\pm 5\%$
t_o	3 in	$\pm 5\%$
E	30 000 ksi	$\pm 5\%$
ν	0.29	$\pm 5\%$
σ_y	26 ksi	$\pm 10\%$
r	5 λt_o	$\pm 10\%$
P	725 000 lb	$\pm 10\%$

$\dagger$ Design Variables

It is important to note that there exists fundamental differences in the manner the deterministic and fuzzy set based approaches maximize the safety measure for a given weight. The deterministic design tends to equalize the two failure loads of Eq. (5.1), while the fuzzy set based design tends to equalize the possibility of failure values due to yielding and due to violating the buckling load constraint. If the design variable limits are not active at the optimum design, one would expect a deterministic design with equal failure load values and a fuzzy set based design with equal possibility of failure values for the two criteria considered in the design.

5.2. Deterministic Design Problem

In the deterministic design, a factor of safety is used to account for the uncertainty in the problem parameters. The safety measure of the structure is thus maximized by maximizing the factor of safety for a given weight. For the deterministic design, failure is calculated from Eqs. (5.1) and (5.2), using the nominal values of the problem parameters, while the non-dimensional stress distribution and buckling load values are obtained from the reduced response surface approximations of CHAPTER 4.

5.2.1. Optimization Problem

The objective of the design problem is to maximize the factor of safety for a given weight. However, since it is difficult to specify a meaningful upper limit of the weight, it was decided to minimize the weight of the plate, for a given factor of safety, in the deterministic design. The resulting minimum weight was then used as the upper limit of the weight for the fuzzy set based design. It was assumed that a factor of safety of 1.5 would be adequate to account for the uncertainty associated with the problem parameters as summarized in Table (5.1). The non-dimensional cross-sectional area $\tilde{A}$ of the plate was used as a representative value of the weight and the resulting optimization problem was defined as:

Minimize:

$$\tilde{A} = \frac{A}{A_o} = \frac{1}{2}(1 + 2\beta + \gamma) + \frac{\lambda}{2}(1 - 2\beta - \gamma) \quad (5.3)$$

Subject to:

$$\begin{aligned} \frac{\beta}{0.4} + 1 &\geq 0 & 1 - \frac{\beta}{0.4} &\geq 0 \\ \gamma &\geq 0 & 1 - \frac{\gamma + \beta}{0.4} &\geq 0 \\ \frac{P_f}{P} - 1.5 &\geq 0 & & \end{aligned} \quad (5.4)$$

The constraints involving β and γ are geometric constraints and P_f is calculated from Eq. (5.1) using the nominal values of the design variables and the reduced stress distribution and buckling load response surface approximations.

5.2.2. Results

The minimum non-dimensional cross-sectional area of the plate, obtained from Eqs. (5.3) and (5.4), are summarized in Table (5.2). For the deterministic optimum design, both the Von Mises yield stress failure criterion and the buckling constraint are active at the optimum design point. The optimum design corresponds to a plate with a change in thickness that starts at the minimum allowable distance from the left endpoint of the plate (see Fig. 5.1) with a very short transition zone (small γ value). Both the possibility of failure values calculated from the vertex method and those predicted by the reduced possibility of failure response surface approximations (values in parenthesis) are summarized in Table (5.2). It is clear that even though both the Von Mises yield stress failure criterion and the buckling load

Table 5.2: Deterministic optimum ($\pm 5\%$ uncertainty in the design variables).

Variable	Value
λ	0.6287
β	-0.4000
γ	0.0447
$\bar{\lambda}^*$	0.6741
Factor of safety (Yield Stress)	1.5
Factor of safety (Buckling Load)	1.5
$\Pi_{\text{YieldStress}}$	0.0977 (0.1181)
Π_{Buckling}	0.3411 (0.3331)

Values in parenthesis are predicted by possibility of failure response surface approximation.

constraint are active at the optimum design point, there exists a large difference between the possibility of failure values for these two criteria, with the buckling load constraint being critical. The accuracy of the possibility of failure response surface approximation is demonstrated, since the difference between the critical predicted and calculated (using the vertex method) possibility of failure values at the optimum design is only 2.3%. The difference between the predicted and calculated possibility of failure values according to the Von Mises yield stress criterion is equal to 20.9%, but yielding is not the active failure mode.

5.3. Fuzzy Set Based Design Problem

The fuzzy set based design problem uses fuzzy set theory to model the uncertainty in the problem parameters. The safety measure of the structure is maximized by minimizing the possibility of failure, using the optimum non-dimensional cross-sectional area obtained from Eqs. (5.3) and (5.4) and summarized in Table (5.2) as an upper limit of the weight. In the fuzzy set based design problem, the possibility of failure is obtained from the reduced possibility of failure response surface approximations of CHAPTER 4.

5.3.1. Optimization Problem

The optimization problem for minimizing the possibility of failure subject to a given weight, may be written as:

Minimize:

$$\Pi_{(P-F_f)} = \hat{\Pi}_{(P-F_f)}(\lambda, \beta, \gamma) \quad (5.5)$$

Subject to:

$$\frac{\beta}{0.4} + 1 \geq 0 \qquad 1 - \frac{\beta}{0.4} \geq 0$$

$$\gamma \geq 0 \quad 1 - \frac{\gamma + \beta}{0.4} \geq 0$$

$$\frac{\tilde{A}(\lambda, \beta, \gamma)}{\tilde{A}^*} - 1 = 0 \quad (5.6)$$

Where $\Pi_{(P-P_f)}$ denotes the possibility of failure, $\hat{\Pi}_{(P-P_f)}$ denotes the approximate possibility of failure obtained from the reduced response surface approximations and $\tilde{A}^*$ denotes the optimum non-dimensional cross-sectional area obtained from the deterministic design. Additionally, the variables λ , β and γ denote the nominal values of the design variables.

5.3.2. Results

The equivalent fuzzy set based design using the $\tilde{A}^*$ value of Table (5.2) is summarized in Table (5.3). The fuzzy set based optimum design corresponds to a plate where the change in thickness starts at the minimum allowable distance from the left endpoint of the plate with no transition zone (γ value equal to 0). The fuzzy set based design eliminates the weight of

Table 5.3: Fuzzy set based optimum ($\pm 5\%$ uncertainty in the design variables).

Variable	Value
λ	0.6379
β	-0.4000
γ	0.0000
$\tilde{A}^*$	0.6741
Factor of safety (Yield Stress)	1.4898
Factor of safety (Buckling Load)	1.5565
$\Pi_{\text{YieldStress}}$	0.1319 (0.1262)
Π_{Buckling}	0.2788 (0.2721)

Values in parenthesis are predicted by possibility of failure response surface approximation.

the ramp and uses it to thicken the thin section of the plate. The result is an increase in the stress concentration and an improvement in the buckling load of the plate. The fuzzy set based design thus attempts to equalize the possibility of failure values due to yielding and due to violating the buckling load constraint by increasing both the stress concentration and the buckling load of the plate. However, in the present design the design variable limits kept the possibility of failure values from becoming equal in the final design. As before, both the possibility of failure values calculated from the vertex method and those predicted from the reduced models of CHAPTER 4 (values in parentheses) are shown in Table (5.3). The difference between the critical predicted and calculated possibility of failure values of the plate at the optimum design is equal to 2.4%. The difference between the predicted and calculated possibility of failure values according to the Von Mises yield stress failure criterion is equal to 4.3%, but again yielding is not the active failure mode.

The membership functions of the possibilities of failure according to the fuzzy function ($P - P_f$) are shown graphically in Fig. (5.2) for the optimum designs obtained from the deterministic and fuzzy set based approaches. The fuzzy function ($P - P_f$) is calculated from Eqs. (5.1) and (5.2), corresponding to the Von Mises yield stress and the buckling load constraint criteria. The membership functions of the optimum deterministic design are shown in solid lines, while the dashed lines indicate the membership functions of the fuzzy set based design. The possibility distributions of Fig. (5.2) illustrate the differences in the manner each approach maximizes a safety measure for a given weight, as discussed in Section 5.1. For the deterministic optimum design, the solid lines converge to a single point for a possibility of failure equal to 1, indicating that the deterministic approach attempts to equalize the factor of safety associated with the Von Mises yield stress failure criterion and that associated with the

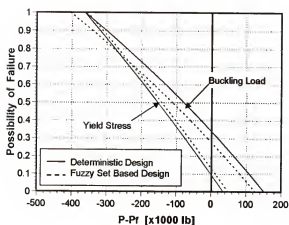


Figure 5.2: Possibility distribution of failure for the deterministic and fuzzy set based optimum designs.

buckling load constraint. For the fuzzy set based optimum design, the dashed lines fall within the solid lines at $(P - P_f) = 0$, indicating that the fuzzy set based approach attempts to equalize the possibility of failure values due to yielding and due to violating the buckling load constraint. However, for the fuzzy set based optimum design the design variable limits kept the two possibility of failure values from becoming equal.

The factor of safety, calculated for the optimum fuzzy set based design, is not much different from that of the deterministic design (only 0.7% lower), however, there is a large difference in the possibility of failure between the two designs. The maximum possibility of failure for the fuzzy set based design is 22.3% lower than that of the deterministic design.

5.4. Cost versus Weight Design Chart

Finally, the dependence of the weight of the structure on the level of uncertainty associated with the design variables was also investigated. For this investigation, the allowable possibility of failure was kept constant at the maximum possibility of failure for the optimum fuzzy set based design obtained from Eqs. (5.5) and (5.6) (i.e., 0.2788 as shown in Table 5.3).

The level of uncertainty associated with the design variables λ , β and γ was varied between $\pm 2\%$ and $\pm 20\%$. As for the fuzzy set based design problem, the reduced possibility of failure response surface approximations of CHAPTER 4 were used to calculate the possibility of failure.

5.4.1. Optimization Problem

For constructing the weight versus cost design chart, seven evenly distributed levels of the uncertainty, u , between $\pm 2\%$ and $\pm 20\%$ were considered. For each of these levels the non-dimensional cross-sectional area of the plate was minimized for the specified allowable possibility of failure. The corresponding optimization problem may then be written as:

Minimize:

$$\tilde{A} = \frac{A}{\lambda_{t_o}} = \frac{1}{2}(1 + 2\beta + \gamma) + \frac{\lambda}{2}(1 - 2\beta - \gamma) \quad (5.7)$$

Subject to:

$$\begin{aligned} \frac{\beta}{0.4} + 1 &\geq 0 & 1 - \frac{\beta}{0.4} &\geq 0 \\ \gamma &\geq 0 & 1 - \frac{\gamma + \beta}{0.4} &\geq 0 \end{aligned}$$

$$\frac{\hat{\Pi}_{(P-F)}(\lambda, \beta, \gamma, u)}{0.2788} - 1 \geq 0 \quad (5.8)$$

5.4.2. Results

Recall that it is assumed that the manufacturing cost of the structure is dominated by the level of uncertainty in the design variables, while the non-dimensional cross-sectional area is representative of the weight. Increasing the level of uncertainty in the design variables corresponds to an increase in the allowable tolerances in these variables and would thus

correspond to a reduction in manufacturing cost. In contrast, an increase in the non-dimensional cross-sectional area would correspond to an increase in weight.

As expected, the non-dimensional cross-sectional area of the plate increased with an increase in the level of uncertainty and the results are shown graphically in Fig. (5.3). Figure (5.3) was constructed based on seven optimizations, indicated by the circular symbols. Figure (5.3) indicates that the increase in weight is almost proportional to the increase in the uncertainty associated with the design variables. The non-dimensional cross-sectional area increased by 10.7% with an 18% increase in the uncertainty associated with the design variables. Using Fig. (5.3) and the dependence of the manufacturing cost on the tolerance of λ , β and γ , the designer may determine what tolerance to use when manufacturing the plate, by trading of manufacturing cost with weight.

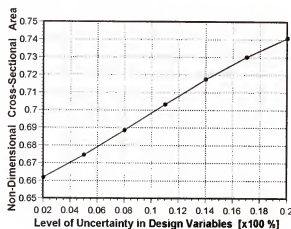


Figure 5.3: Non-dimensional cross-sectional area compared to the level of uncertainty in the design variables.

5.5. Reduction in Computational Cost

Response surface approximations formed an integral part of the fuzzy set based design process presented here. The reduced stress distribution and buckling load response surface

approximations of CHAPTER 4 were used to replace computationally expensive finite element analyses during the evaluation of the possibility of failure. Fuzzy set based designing for uncertainty is an expensive two level optimization problem and the use of the response surface approximations resulted in substantial reductions in computational costs due to the following two reasons:

1. It is cheaper to construct all of the required approximations than to perform a single two level optimization problem.
2. The computational cost is shifted from the optimization problem to that of constructing the approximations. Shifting the computational cost to the problem of constructing the approximations allows the use of the approximations in additional optimization problems at very little additional computational cost.

The reductions in computational cost are illustrated for a single two level optimization problem, by considering all problem parameters to be uncertain, as is the case in the present design problem. The evaluation of the possibility of failure for a single α level cut then requires $2 \cdot 2^8 = 512$ finite element analyses when no response surface approximations are used. For a single optimization, a conservative estimate of the required number of finite element analyses required, when not using response surface approximations, is obtained from the product of the following four numbers:

Average number of design optimization iterations:	5
Average number of $\Pi_{(P,P_f)}$ evaluations per iteration:	6
Average number of α level cut evaluations per $\Pi_{(P,P_f)}$ evaluation:	5
Number of finite element analyses per α level cut evaluation of $\Pi_{(P,P_f)}$:	512
Total number of finite element analyses required per optimization:	<u><u>76,800</u></u>

In contrast, the stress distribution and buckling load response surface approximations were constructed, and evaluated, from a total of only 752 finite element analyses. These results indicate that response surface approximations provide an effective approach for reducing the computational cost associated with performing fuzzy set based design. The large number of computationally expensive finite element analyses required to perform the fuzzy set based design is replaced by response surface approximations that are inexpensive to evaluate.

By using response surface approximations, the computational burden shifts from the optimization problem to the problem of constructing the response surface approximations. Due to the iterative nature of the design process, the fact that response surface approximations allow multiple optimizations at minimal additional computational cost should be an attractive feature to any designer. Additionally, the fact that multiple optimizations may be performed at little additional computational cost allows the designer to investigate, for example, the dependence of the weight on the uncertainty in the design variables in the form of weight versus cost design charts. The reduced possibility of failure response surface approximation was used mainly to simplify the integration of the analysis code and the optimization algorithm, but also to filter out numerical noise by providing an approximate response function, thus allowing the use of a derivative-based optimization algorithm.

CHAPTER 6

DROPPED PLY COMPOSITE LAMINATED PLATE RESPONSE SURFACE CONSTRUCTION

Designing for uncertainty of a composite laminated plate with internally terminated plies (referred to as dropped plies) is the second example problem considered. The dropped plies result in a change in thickness across the width of the plate and are used to reduce the weight of the structure. The weight is reduced by reducing the thickness according to the loads so that the structure is thicker in regions with higher loads. It is assumed that the structure will be built from future materials, with little information available on uncertainty in the material properties, which will be modeled by fuzzy set theory. The present problem is thus an extension of the isotropic plate example problem, where the complexity of the problem is increased by considering a composite laminated plate. For the present problem the material properties and applied load are considered to be uncertain. Additionally, manufacturing tolerances are also modeled by considering the uncertainty in the lay-up angle of the individual plies of the composite laminated plate. The design optimization trades off costs associated with material property characterization versus costs associated with increased structural weight.

6.1. Problem Description

Reducing the weight by reducing the thickness of composite laminated structures according to the loads is common in the aircraft industry. Examples include helicopter rotor blades as well as wing structures, where the thickness is reduced from the root to the tip. The

internal termination of plies is an effective method for reducing the thickness of composite laminated structures. Unfortunately, dropped plies result in severe stress concentrations due to the stiffness and thickness discontinuities associated with the internal termination of the plies. These stress concentrations may greatly reduce the strength of the structure, thus offsetting the advantages of reducing the thickness by terminating plies.

The dropped ply problem has been studied extensively, and a large number of analytical, numerical and experimental studies has been published since the mid 1980's. For example Kemp and Johnson⁷⁹ (1985) performed a numerical study on graphite-epoxy composite laminated plates subjected to both a tensile and a compressive applied load, while Fish and Lee⁸⁰ (1988) performed a numerical and experimental study on tensile loaded glass-epoxy composite laminated plates. Dinardo and Lagace⁸¹ (1989) performed both an analytical and experimental study of the buckling and post-buckling behavior of graphite-epoxy composite laminated plates, while Curry, Johnson and Starnes⁸² (1992) performed a numerical and experimental study of similar structures subjected to both a tensile and a compressive applied load. Additionally, Llanos and Vizzini⁸³ (1992) investigated the influence of adding film adhesive on the strength of tension loaded glass-epoxy specimens, while Fish and Vizzini^{84,85} (1992 and 1993) Botting, Vizzini and Lee⁸⁶ (1996) studied the influence of changing the ply drop configuration. Dávila and Johnson⁸⁷ (1993) performed a combination of numerical and experimental analyses of the buckling and post-buckling behavior of graphite-epoxy composite laminated plates and Harrison and Johnson⁸⁸ (1993) and Rose and Starnes⁸⁹ (1996) formulated efficient approximate analyses for determining the inter-laminar stresses in composite laminated plates with thickness discontinuities. Finally, Vizzini⁹⁰ (1995) studied the influence of including imperfections that naturally occur during the manufacturing process, in

the modeling of glass-epoxy structures. The experimental studies indicated that delamination is a common mode of failure. As a result, all of the above mentioned studies concentrated on failure initiation in the form of delamination that occurs in the interfaces directly above and below the dropped plies. The studies indicated that the presence of dropped plies greatly reduces the strength of the resulting composite laminated plate.

In the present example problem, the cost design of a composite laminated plate with dropped plies is considered. The plate structure is typical of wing skin applications, where aerodynamic requirements for a smooth surface introduce asymmetry. This problem was studied extensively by Curry, Johnson and Starnes⁸² (1992) and by Dávila and Johnson⁸⁷ (1993) and is shown schematically in Fig (6.1).

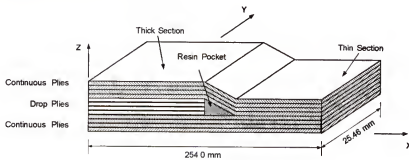


Figure 6.1: Dropped ply composite laminated plate typical of wing skin applications.

Curry, Johnson and Starnes⁸² (1992) performed an experimental study, considering 37 specimens that differed only in the configuration of the dropped plies. The experimental study revealed that the axial strength of the dropped ply composite laminated plate considered is less than that of the thin section, and that the compressive strength is less than the tensile strength. The authors also concluded that the reduction in axial strength is directly related to the change in axial stiffness between the thick and the thin sections.

Dávila and Johnson⁸⁷ (1993) correlated the experimental results of Ref. (82) with detailed finite element analyses and proposed a delamination initiation failure index based on an existing quadratic inter-laminar failure index. The proposed failure index is evaluated at a characteristic distance from the ply drop stress singularity and correlated well with the failure initiation observed in the experimental data of Ref. (82). However, the success of the method depends on the availability of experimental data for determining the characteristic distance.

The present problem is based on the 254.0 mm by 25.46 mm, AS/3502 graphite-epoxy (material properties summarized in Table 6.1) plate shown in Fig. (6.1), that was considered by Curry, Johnson and Starnes⁸² (1992) and by Dávila and Johnson⁸⁷ (1993). However, in the present problem the plies are dropped in two steps, separated by 20 ply thicknesses, and the

Table 6.1: Composite laminated plate material properties.

AS4/3502 Graphite-Epoxy		
E_1	126 GPa	Young's modulus in fiber direction
E_2	11.3 GPa	Young's modulus in matrix direction
E_3	11.3 GPa	Young's modulus in thickness direction
ν_{12}	0.3	Poisson's ratio in 1-2 plane
ν_{13}	0.3	Poisson's ratio in 1-3 plane
ν_{23}	0.35	Poisson's ratio in 2-3 plane
G_{12}	6.0 GPa	Shear modulus in 1-2 plane
G_{13}	6.0 GPa	Shear modulus in 1-3 plane
G_{23}	3.38 GPa	Shear modulus in 2-3 plane
Neat Resin		
E	3.45 GPa	Young's modulus
ν	0.41	Poisson's ratio
G	1.34 GPa	Shear modulus
Strength Allowables		
X_t	1.45 GPa	Fiber direction tensile strength
X_c	-1.45 GPa	Fiber direction compressive strength
Y_t, Z_t	52 MPa	Transverse tensile strength
Y_c, Z_c	-206 MPa	Transverse compressive strength
S_{12}, S_{13}, S_{23}	93.1 MPa	Shear strengths

resin pockets are reinforced with 90° fibers, measure relative to the length direction of the plate (i.e., the x-direction). The present problem is shown schematically in Fig. (6.2). The drop zone geometry used in the present problem was determined based on an investigation using finite element analyses as discussed in Appendix A.

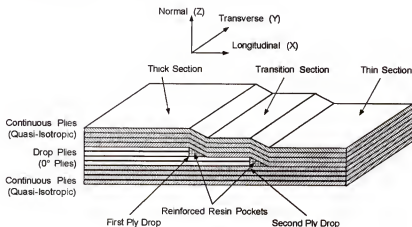


Figure 6.2: Two step dropped ply composite laminated plate.

As shown in Fig. (6.2), the laminated plate consists of three sub-laminates where four 0° plies are dropped and the sub-laminates surrounding the dropped plies are quasi-isotropic. More specifically, the lay-up of the three sub-laminates in the thick section of the plate is $[(\pm 45/0/90)_s(0_2)_s(\pm 45/0/90)_s]_T$. The three sub-laminates are reduced to two sub-laminates in the thin section, with a lay-up of $[(\pm 45/0/90)_s(\pm 45/0/90)_s]_T$. A detailed cross-section of the dropped ply composite laminated plate is shown in Fig. (6.3).

Two failure mechanisms were considered in the design, delamination in the interfaces directly above and below the dropped plies, and ply failure. Response surface approximations of the stress and strain components required to evaluate these failure mechanisms (discussed subsequently in Section 6.2) were constructed and used during the calculation of the possibility of failure of the composite laminated plate. These response surface approximations

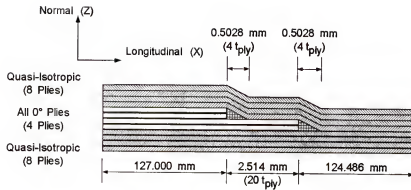


Figure 6.3: Cross-section model with dimensions shown.

were constructed from numerical experiments in the form of linear finite element analyses with CSAR/NASTRAN Version 97⁹¹ (1997).

Due to symmetry, only half the width (i.e., y-direction) of the plate was modeled and the finite element model had a total of 2,814 elements and 4,020 nodes. The model consisted of a combination of four-node plate bending elements and eight-node solid elements as shown schematically in Fig. (6.4). Multi-point constraints were used to connect the plate bending elements (with rotational degrees of freedom) to the solid elements (without rotational degrees of freedom).

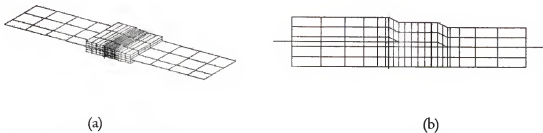


Figure 6.4: Schematic finite element model (actual model had 42 two-dimensional and 2,772 three-dimensional elements): (a) Overall perspective; (b) Detailed perspective near the ply drop.

The four-node plate bending elements were used to model the plate away from the ply drop, while the eight-node solid elements were used for the detailed modeling of a small region near the ply drop. Three dimensional elements are required in the vicinity of the ply drop in order to model the three dimensional stress field in this region. Although multiple three dimensional elements were used through the thickness of each sub-laminate, these elements do not correspond to the ply lay-up of the sub-laminates. Instead each sub-laminate was defined in terms of its equivalent homogeneous orthotropic material properties. The equivalent homogeneous orthotropic material properties were calculated using the methodology as outlined in Sun and Li⁹² (1988).

The boundary conditions of the finite element model were chosen to approximate the experimental setup used by Curry, Johnson and Starnes⁸² (1992) and are shown schematically in Fig. (6.5). A compressive load of 507.3 N/mm was applied at the thick end (i.e., $x=0$ mm) where only a uniform displacement in the x -direction was allowed (all other degrees of freedom were fixed). At the thin end (i.e., $x=254.0$ mm) all six degrees of freedom were fixed.

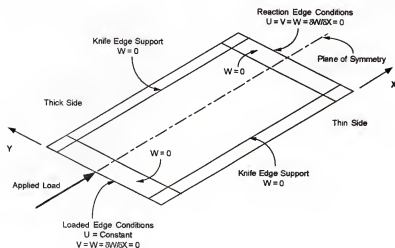


Figure 6.5: Schematic representation of the boundary conditions.

Additionally, the out of plane displacements were set equal to zero within a 9.525 mm region along the width of each endpoint (i.e., $x=0$ mm and $x=254.0$ mm). Along the length of the plate, the structure was supported by two knife edges applied 3.175 mm inward from the free edges (i.e., $y=0$ mm and $y=25.46$ mm). For the shell elements, the knife edges were modeled by setting the out of plane displacement equal to zero. For the solid elements, the knife-edges were modeled by setting the out of plane displacement equal to zero along the bottom of the plate (i.e., $z=0$ mm). Additionally, a small pinching displacement equal to $-1 \times 10^{-4} t_o$, where t_o is the thickness of the thick side of the plate, was imposed along the top surface of the thick section of the plate. This pinching displacement was used to simulate the clamping action of the knife edge support.

6.2. Analysis Issues

The baseline structure (nominal values of problem parameters as shown in Table 6.1) was investigated in order to identify the critical region for each of the delamination and ply failure mechanisms. Response surface approximations of the stress and strain components required to evaluate the failure mechanisms were then constructed in the critical regions identified by this investigation.

Based on work by Whitney and Nuismer⁹³ (1974), a characteristic distance was used to account for the stress singularities, and both the delamination and ply failure criteria were evaluated at a characteristic distance from the corresponding stress concentration. The characteristic distance must be determined experimentally. The results of Dávila and Johnson⁹⁷ (1993) indicated that the characteristic distance depends on both the material used and the lay-up of the composite laminated plate. Due to a lack of experimental data in the present dissertation, it was assumed that both failure criteria are evaluated at an equal

characteristic distance of 0.25 mm. This value of the characteristic distance was based on results obtained by Dávila and Johnson⁸⁷ (1993) for the problem shown in Fig. (6.1). Note that neither of the assumptions (i.e., equal characteristic distance for the two failure modes and a characteristic distance of 0.25 mm) restricts the generality of the proposed methodology. The response surface approximations of the stress and strain distributions, used to calculate the possibility of failure, do not depend on the characteristic distance.

6.2.1. Ply Failure

The maximum stress failure criterion (Gibson⁹⁴ 1994, pp. 103-106) was used to find the most critical region for ply failure. The maximum stress criterion predicts failure when any stress component in the principal material system exceeds the corresponding strength, and thus to avoid failure the following inequalities must be satisfied

$$X_c < \sigma_1 < X_t, \quad Y_c < \sigma_2 < Y_t, \quad |\sigma_{12}| < S_{12} \quad (6.1)$$

where the subscript 1 indicates the longitudinal fiber direction and the subscript 2 indicates the transverse fiber direction as shown in Fig. (6.6). Additionally, the subscript *c* denotes compression and the subscript *t* denotes tension.

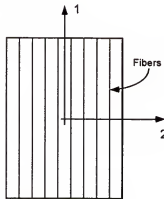


Figure 6.6: Composite laminated plate fiber directions.

The maximum stress criterion must be applied to each ply since, unlike homogenous, isotropic materials, it is not obvious which is the critical ply. In the present dissertation, the topmost ply on the thinnest section of the laminated plate was identified as the critical ply. This ply corresponds to the -45° ply ahead of the second ply drop as shown in Fig. (6.2). For this ply σ_1 and σ_2 are in compression and the failure indices, based on the maximum stress criterion of Eq. (6.1) are defined as:

$$F_1 = \frac{\sigma_1}{X_c} < 1 \quad F_2 = \frac{\sigma_2}{Y_c} < 1 \quad F_{12} = \frac{|\sigma_{12}|}{S_{12}} < 1 \quad (6.2)$$

Failure thus occurs if any of the failure indices of Eq. (6.2) is equal to or larger than 1, with higher failure index values indicating more critical stress components. The failure indices calculated from the baseline problem, with no uncertainty in the problem parameters, are shown graphically in Fig. (6.7) for the topmost ply of each of the two thinner sections of the composite laminated plate. These thinner sections correspond to the first and the second ply drops as shown in Fig. (6.2).

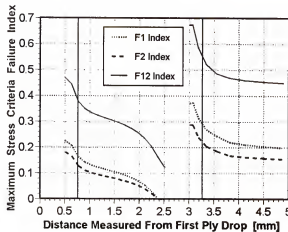


Figure 6.7: Ply failure indices for the topmost ply of the two thinner sections.

In Fig. (6.7) the x -axis represents a local coordinate system with origin at the first ply drop and the solid vertical lines indicate the characteristic distance. Figure (6.7) indicates that the shear stress component on the top surface of the thinnest section of the dropped ply composite laminated plate is the most critical stress component. To be more consistent with the convention used by industry, the maximum strain failure criterion was used throughout the design process, by converting the shear stress allowable to a strain allowable given by:

$$e_{12} = \frac{S_{12}}{G_{12}} \quad (6.3)$$

The critical ply failure index F_{ply} of the structure may then be written in terms of the γ_{12} shear strain on the top surface of the thinnest section of the laminated plate and the allowable shear strain e_{12} as follows:

$$F_{ply} = \frac{\gamma_{12}}{e_{12}} \quad (6.4)$$

The ply failure load P_{ply} is the defined as

$$P_{ply} = \frac{P}{F_{ply}} \quad (6.5)$$

where P denotes the applied load. This failure index is evaluated at the characteristic distance away from the re-entrant corner.

6.2.2. Delamination Failure

For the delamination failure, the failure index proposed and successfully evaluated by Dávila and Johnson⁸⁷ (1993) was used. This failure index is based on a quadratic failure index introduced by Brewer and Lagace⁹⁵ (1988)

$$F_{Delam}^2 = \left(\frac{\sigma_{ZX}}{S_{13}} \right)^2 + \left(\frac{\sigma_{ZY}}{S_{23}} \right)^2 + \left(\frac{\sigma_{ZZ}}{Z_t} \right)^2 \quad \text{where } \sigma_{ZZ} > 0 \quad (6.6)$$

where σ_{zx} , σ_{zy} and σ_{zz} are the inter-laminar stress components and S_{12} , S_{23} and Z_t are the inter-laminar strength allowables (see Table 6.1). This failure index was evaluated in the interfaces directly above and below the dropped plies, at a characteristic distance away from the ply drop. Additionally, in order to avoid free edge failure, the failure index was only evaluated inwards from the knife edge support shown in Fig. (6.5). The delamination failure load P_{Delam} is calculated from the failure index as:

$$P_{Delam} = \frac{P}{F_{Delam}} \quad (6.7)$$

To identify the critical region for delamination failure, the delamination failure index F_{Delam} was evaluated for the baseline problem with no uncertainty in the problem parameters and the results are shown graphically in Fig. (6.8). In Fig. (6.8) the x-axis represents a local coordinate system with origin at the first ply drop and the solid vertical lines indicate the characteristic distance. Figure (6.8) indicates that the interface above the second step of the ply drop corresponding to the transition section as shown in Fig. 6.2 is the most critical.

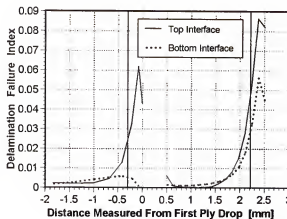


Figure 6.8: Delamination failure index above and below the dropped plies.

6.3. Dimensional Analysis

As for the isotropic plate example problem, dimensional analysis was used to reduce the number of predictor variables of the stress and strain distribution response surface approximations, where the predictor variables are the variables used in the response surface approximations. For the present example problem, the material properties, characteristic distance, lay-up angles and applied load were considered as uncertain and the stress and strain distributions were thus required as a function of these uncertain problem parameters. Unlike the isotropic plate example problem, the predictor variables of the present problem were thus known beforehand and the Buckingham Π Theorem (see Appendix B) could be used to non-dimensionalize the problem, using the functional form of the stress and strain distributions.

The stress and strain equations may be written in functional form in terms of the material properties, the geometry and the boundary conditions and applied load. The fundamental dimensional units of both the stress and strain distributions are force [F] and length [L]. Thus, both the stress and strain distributions may be written in non-dimensional form by selecting E_l (dimension $[F.L^{-2}]$) and a dummy variable l of unit length (dimension [L]) as the predictor variables with independent dimensions.

Since a linear finite element analysis is performed, both the stress and the strain distributions depend linearly on the magnitude of the applied load. The applied load can thus be eliminated as a predictor variable, since the stress and strain distributions can simply be scaled for different values of the applied load as follows:

$$\gamma = \gamma_o \frac{P}{P_o} \qquad \sigma = \sigma_o \frac{P}{P_o} \qquad (6.8)$$

In Eq. (6.8) P_o is the applied load used in the liner finite element analysis and σ_o and γ_o are the corresponding stress and strain values, while P represents a different value of the applied load and σ and γ are the corresponding stress and strain values.

Additionally, because equivalent orthotropic material properties were used in the modeling of each sub-laminate, the number of variables required to characterize the lay-up of each sub-laminate may be greatly reduced by making use of lamination parameters. Using lamination parameters, the lay-up of each sub-laminate may be characterized by only two non-dimensional lamination parameters (Haftka and Gürdal¹⁸ pp. 415-429) instead of the lay-up angle of each ply, as follows

$$V_1^* = \frac{1}{n} \sum_{i=0}^n \cos(2\theta_i) \quad V_3^* = \frac{1}{n} \sum_{i=0}^n \cos(4\theta_i) \quad (6.9)$$

where it is assumed that all plies have the same thickness. Additionally, n denotes the total number of plies in the sub-laminate and θ_i the lay-up angle of the i^{th} ply.

Based on the above observations, the non-dimensional functional relationships of the stress and strain distributions may be written in terms of the predictor variables as

$$\begin{aligned} \gamma_{12} &= \gamma_{12} \left(\frac{E_2}{E_1}, \frac{G_{12}}{E_1}, \frac{G_{23}}{E_1}, \nu_{12}, \nu_{23}, V_{1\_Drop}^*, V_{3\_Drop}^*, V_{1\_Top}^*, V_{3\_Top}^*, V_{1\_Bottom}^*, V_{3\_Bottom}^*, \frac{x}{l} \right) \\ \frac{\sigma_{zz}}{E_1} &= \frac{\sigma_{zz}}{E_1} \left(\frac{E_2}{E_1}, \frac{G_{12}}{E_1}, \frac{G_{23}}{E_1}, \nu_{12}, \nu_{23}, V_{1\_Drop}^*, V_{3\_Drop}^*, V_{1\_Top}^*, V_{3\_Top}^*, V_{1\_Bottom}^*, V_{3\_Bottom}^*, \frac{x}{l} \right) \\ \frac{\sigma_{zx}}{E_1} &= \frac{\sigma_{zx}}{E_1} \left(\frac{E_2}{E_1}, \frac{G_{12}}{E_1}, \frac{G_{23}}{E_1}, \nu_{12}, \nu_{23}, V_{1\_Drop}^*, V_{3\_Drop}^*, V_{1\_Top}^*, V_{3\_Top}^*, V_{1\_Bottom}^*, V_{3\_Bottom}^*, \frac{x}{l} \right) \\ \frac{\sigma_{zy}}{E_1} &= \frac{\sigma_{zy}}{E_1} \left(\frac{E_2}{E_1}, \frac{G_{12}}{E_1}, \frac{G_{23}}{E_1}, \nu_{12}, \nu_{23}, V_{1\_Drop}^*, V_{3\_Drop}^*, V_{1\_Top}^*, V_{3\_Top}^*, V_{1\_Bottom}^*, V_{3\_Bottom}^*, \frac{x}{l} \right) \end{aligned} \quad (6.10)$$

where the subscripts *Drop*, *Top* and *Bottom* are used to denote the respective sub-laminates.

Additionally, it was assumed that $E_2=E_3$, $G_{12}=G_{31}$ and $\nu_{12}=\nu_{13}$ as shown in Table (6.1).

Using the non-dimensional lamination parameters, the number of predictor variables required to characterize the lay-up of each sub-laminate is greatly reduced from the lay-up angle of each ply (total of 20 variables) to only two lamination parameters per sub-laminate (total of 6 variables). Additionally, the remaining predictor variables were reduced from 14 to 12 by making use of dimensional analysis and the fact that the stress and strain distributions depend linearly on the applied load.

6.4. Calculating the Possibility of Failure

The possibility of failure is calculated from the load margin ($P - P_f$) of the structure, with failure defined to occur when

$$P - P_f \geq 0 \quad (6.11)$$

and the failure load P_f is calculated from the ply and delamination failure loads as follows:

$$P_f = \min \begin{cases} P_{Ply} \\ P_{Delam} \end{cases} \quad (6.12)$$

For the present example problem, the design space is not a rectangular domain in all of the design variables. In particular, the design space for the lamination parameters V_1^* and V_3^* of the top and bottom sub-laminates are non-rectangular and are shown graphically in Fig. (6.9) for a 5 degree uncertainty in the ply lay-up angle.

Note that there exists a linear relationship between $V_{1\_Drop}^*$ and $V_{3\_Drop}^*$. The total number of predictor variables is thus reduced from 12 to only 11 by eliminating $V_{3\_Drop}^*$ as a predictor variable. Apart from the V_1^* and V_3^* design spaces of the top and bottom sub-laminates that are non-rectangular, both the $V_{1\_Drop}^*$ design space and the V_1^* and V_3^* design spaces for the top and bottom sub-laminates are also discrete.

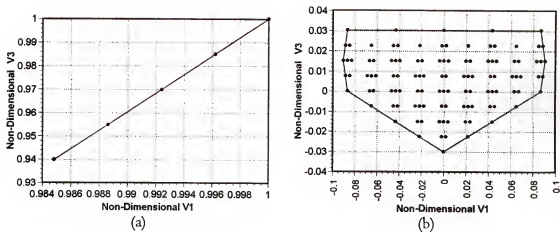


Figure 6.9: All possible combinations of the lamination parameters V_1^* and V_3^* : (a) Dropped ply sub-laminate; (b) Bottom and top sub-laminates.

The Design Optimization Tools (DOT)⁷⁰ (1995) software was used to verify that the global optimum of the load margin ($P - P_f$) (see Eq. 6.11) occurred at a vertex of the design space, thus validating the use of the vertex method to calculate the possibility of failure. Since the sequential linear programming algorithm provided with the Design Optimization Tools (DOT)⁷⁰ (1995) software converges on a local minimum in the design space, 1,000 optimizations were performed from different starting points randomly distributed throughout the design space to ensure that a global optimum is found. These optimizations were performed for the zero α level cut using the maximum level of uncertainty associated with each design variable (shown subsequently in Table 6.3). The best solutions obtained from the 1,000 optimizations are summarized in Table (6.2). For the delamination load margin of Eq. (6.11), the overall minimum value was found on the boundary of the V_1^* and V_3^* design spaces for the top and bottom sub-laminates, but not at a vertex. Since the V_1^* and V_3^* design spaces for the top and bottom sub-laminates are discrete (see Fig. 6.9), all 20 boundary points for each of the two V_1^* and V_3^* design spaces were included in the vertex method to ensure that a global optimum is found. The results obtained from the vertex method using only the

Table 6.2: Maximal load margins for the zero α level cut.

Optimization Method	$P - P_{Ply}$	$P - P_{Delam}$
Design Optimization Tools Software	-4,130.79 N	-6,351.83 N
Vertex Method (Vertices Only)	-4,164.83 N	-6,126.35 N
Vertex Method (All Boundary Points)	-4,164.83 N	-6,365.14 N

seven vertex points for each of the V_1^* and V_3^* design spaces for the top and bottom sub-laminates, as well as the results obtained from the vertex method using all 20 boundary points are also summarized in Table (6.2).

Note that the sequential linear programming algorithm provided with the Design Optimization Tools (DOT)⁷⁰ (1995) software is a continuous optimization algorithm, while the vertex method considers only a discrete number of design points. However, the vertex method gave slightly better results than the sequential linear programming algorithm (0.82% better for $P - P_{Ply}$ and 0.21% for $P - P_{Delam}$) since, as expected, the sequential linear programming algorithm converged on local minima in the design space. The optimum results for $(P - P_{Delam})$ obtained by the sequential linear programming algorithm are shown graphically in Fig. (6.10) for the first 100 of the original 1,000 optimizations. Ten of the 100 optimizations did not satisfy all of the constraints within the allowable number of design iterations, which was set at 500 iterations due to computational constraints. Thus, Fig. (6.10) only shows the optimum results of the remaining 90 optimizations. The results shown in Fig. (6.10) have a 26.72% variation between the minimum and maximum $(P - P_{Delam})$ optimum values, indicating the need for an expensive global optimization algorithm, like the vertex method, to calculate the possibility of failure accurately.

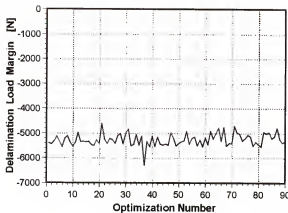


Figure 6.10: Optimal results obtained by the Design Optimization Tools software for $(P - P_{Delam})$.

6.5. Response Surface Approximation Construction

As for the isotropic plate example problem, response surface approximations were used mainly to reduce the computational burden associated with calculating the possibility of failure. For the dropped ply composite laminated plate problem, the failure load P_f is calculated from (as shown in Section 6.4)

$$P_f = \text{Min} \begin{cases} P_{Ply} \\ P_{Delam} \end{cases} \quad (6.13)$$

and failure is defined to occur when:

$$P - P_f \geq 0 \quad (6.14)$$

It was further assumed that delamination represents catastrophic failure corresponding to the ultimate load, while ply failure only represents the onset of failure corresponding to the limit load. In the aircraft industry the ultimate load is typically assumed to be 1.5 times greater than the limit load, which is incorporated as a factor of safety for the delamination load, while a factor of safety equal to 1.0 was used for the ply failure load.

As shown in the flow diagram of Fig. (4.5), two levels of response surface approximations were constructed. On the first level, accurate response surface approximations were constructed for evaluating the ply failure and delamination failure loads. These approximations were then used instead of computationally expensive finite element analyses during the calculation of the possibility of failure based on Eqs. (6.13) and (6.14). On the second level, response surface approximations were constructed for the possibility of failure as a function of the level of uncertainty associated with the uncertain problem parameters. This approximation was constructed mainly to simplify the integration of the analysis code and the optimization algorithm. However, due to the relatively large number of uncertain problem parameters in the present example problem, calculating the possibility of failure from the delamination and ply failure response surface approximations is computationally intensive in itself. The second level response surface approximation thus also results in some reduction in computational cost.

In constructing the second level response surface approximation for the possibility of failure, the predictor variables were obtained by dividing the uncertain problem parameters into five groups, where all the parameters in a specific group have the same level of uncertainty. The five groups are the elastic material properties, the ply and delamination failure strength allowables, the ply lay-up angle, the characteristic distance and the applied load. The predictor variables were defined as the uncertainty associated with the elastic material properties (EL), the ply failure strength allowable (SP), the ply lay-up angle (PL) and the characteristic distance (CD). The uncertainty associated with the delamination strength allowables (SD) was also considered, but not as an independent variable. Instead, the uncertainty in the delamination strength allowables was assumed to be twice that associated

with the ply failure strength allowable. Additionally, the weight W of the structure was considered as a fifth predictor variable in the possibility of failure response surface approximation.

The uncertain problem parameters and the nominal level of uncertainty associated with each parameter are summarized in Table (6.3).

Table 6.3: Uncertain problem parameters.

Property	Description	Uncertainty	Group
E_1	Young's Modulus	10%	Elastic Material Properties (EL)
E_2, E_3	Young's Modulus	10%	
G_{12}, G_{13}	Shear Modulus	10%	
G_{23}	Shear Modulus	10%	
ν_{12}, ν_{13}	Poisson's Ratio	10%	
ν_{23}	Poisson's Ratio	10%	
e_{12}	Allowable transverse shear strain	20%	Ply Failure Strength Allowable (SP)
S_{12}	Allowable transverse shear strength [†]	40%	
Z_t	Allowable transverse tensile strength [†]	40%	
θ_i	Lay-up Angle ^{††}	5°	Ply Lay-Up (PL)
P	Applied load	20%	Load
d_o	Characteristic Distance	20%	Characteristic Distance (CD)

[†] The uncertainty associated with the delamination strength allowables are assumed to be twice as large as that associated with the ply failure strength allowable.

^{††} The uncertainty in the lay-up angle and the original lay-up of each sub-laminate is used to calculate V_1^* and V_3^* for the dropped, bottom and top sub-laminates respectively.

6.5.1. Stress and Strain Distribution Response Surface Approximations

The stress and strain distribution response surface approximations have a relatively large number of predictor variables (11 predictor variables) and the computational cost associated with evaluating a single finite element analysis is high. Therefore, statistical design of experiments, in particular the D -Optimality criterion, was used to find a small set of data points for constructing the stress and strain distribution response surface approximations

without compromising accuracy. The *D*-Optimality criterion was selected since the design space is not a rectangular or sphere-like shape in all the design variables (see Fig. 6.9), and the standard designs (e.g., Box-Behnken and Central Composite designs) are thus not applicable.

A full quadratic response surface approximation with 78 parameters was assumed as an initial response surface approximation for the stress and strain distributions. Note that each laminate configuration (corresponding to an individual finite element model) is described by only 10 of the 11 predictor variables. Each finite element model includes a number of data points with different x/l values (the 11th predictor variable) corresponding to different finite elements. The *D*-Optimality design criterion was used to select 200 data points, corresponding to 200 finite element analyses, for the 10 predictor variables that influence the configuration of the composite laminated plate. For the full quadratic response surface approximation with 78 parameters, the number of data points were thus chosen to be roughly equal to two and a half times the number of parameters in the response surface approximation. The number of data points identified by the *D*-Optimality criterion is typically selected to be between one and a half and three times the number of parameters in the response surface approximation. The ratio of the number of data points to the number of parameters is heavily influenced by the cost of evaluating the response at a single data point. The *D*-Optimal data points were selected from a set of 57,049 candidate points that were obtained by considering a three level factorial design in the 10 relevant predictor variables, consisting of three evenly spaced levels for each predictor variable. An additional 50 data points, randomly distributed throughout the design space, were chosen for evaluating the predictive capabilities of the response surface approximations.

For each finite element analysis, the stress and strain values at all node points within 1 mm of the relevant stress concentration (i.e., the ply drop for delamination failure and the re-entrant corner for ply failure) were recorded. These stress and strain values were used in constructing the response surface approximations in terms of the 11th predictor variable, x/l . In constructing the response surface approximations, the node points that coincide with the stress singularities (i.e., the node points at the ply drop and the re-entrant corner) were ignored in order to avoid numerical inaccuracies associated with the finite element models at these node points. The finite element models are inaccurate at the stress singularities since the stress and strain values tend to infinity, which cannot be represented accurately by a finite element analysis. Ignoring the node points at the stress singularities implies that the response surface approximations would only predict realistic stress and strain values some distance away from the stress singularities, and not at the singularities themselves. Since the characteristic distance used in evaluating both failure criteria spans a number of node points in the finite element model, the response surface approximations were sufficient to obtain accurate stress and strain values. This process resulted in 1,000 data points for constructing the γ_{12} strain distribution and 1,200 data points for constructing each of the σ_{zz} , σ_{zx} and σ_{zy} stress distribution response surface approximations (Data Set 1). Additionally, 250 independent data points were available for evaluating the predictive capabilities for the γ_{12} strain distribution, and 300 independent data points were available for evaluating the predictive capabilities for each of the σ_{zz} , σ_{zx} and σ_{zy} stress distribution response surface approximations (Data Set 2).

The initial quadratic response surface approximation was used to identify the most important variables for describing the response. Each variable was varied between its upper and lower bounds with all other variables kept constant and the resulting influence on the

response function was investigated. The variables with the largest influence on the response function were identified as the most important variables. This investigation was performed graphically using JMP⁶⁷ (1995), which allows the user to easily study the influence on the response when changing a single variable for different levels of the remaining variables. Four variables were identified as important variables and all cubic parameters in these variables were added to the initial quadratic response surface approximation. Adding these cubic terms increased the number of parameters in the initial response surface approximation from 78 to 98. Note that a full cubic polynomial would result in a response surface approximation with 364 parameters. Due to the relatively high computational cost associated with evaluating the response at a single data point, evaluating the response at the large number of data points required for constructing a response surface approximation with 364 parameters was impractical.

The *D*-Optimal data points were selected based on the original quadratic response surface approximation and should thus be used with caution in constructing the enhanced response surface approximation that includes both quadratic and cubic parameters. For the present example problem, the number of *D*-Optimal data points is roughly equal to two times the total number of parameters in the enhanced model. Additionally, the random points used for evaluating the predictive capabilities of the response surface approximations would indicate when the selected *D*-Optimal data points are not appropriate for constructing the enhanced response surface approximation. When the selected data points are not appropriate for constructing the enhanced response surface approximation, the error terms associated with the random data points would be much greater than the error terms associated with the data point used to construct the response surface approximation.

6.5.1.1. Strain distribution response surface approximation (Ply failure)

The ply failure load may be calculated from Eqs. (6.4) and (6.5), and depends only on the γ_{12} strain component on the top surface of the thinnest section of the composite laminated plate. The γ_{12} strain component may be written in non-dimensional functional form (see Eq. 6.10) as:

$$\gamma_{12} = \gamma_{12} \left(\frac{E_2}{E_1}, \frac{G_{12}}{E_1}, \frac{G_{23}}{E_1}, u_{12}, u_{23}, V_{1\_Drop}^*, V_{1\_Top}^*, V_{3\_Top}^*, V_{1\_Bottom}^*, V_{3\_Bottom}^*, \frac{x}{l} \right) \quad (6.15)$$

Where $V_{3\_Drop}^*$ depends linearly on $V_{1\_Drop}^*$ and is not required as a predictor variable (see Section 6.5)

For the γ_{12} strain component, the quadratic response surface approximation was enhanced by including all third order parameters for the four most important variables, which were identified as: E_2/E_1 , G_{12}/E_1 , $V_{1\_Drop}^*$ and x/l . The number of parameters in the enhanced response surface approximation was reduced from 98 to 60 using the stepwise regression procedure and the C_p statistic. Using both Data Set 1 and Data Set 2, the predictive capabilities of the full quadratic response surface approximation (78 parameters), the full enhanced response surface approximation (98 parameters), and the reduced enhanced response surface approximation (60 parameters) were compared and the results are summarized in Table (6.4).

Table (6.4) indicates that the enhanced response surface approximations are more accurate than the original quadratic response surface approximation, with the reduced enhanced response surface approximation slightly more accurate (except for the %RMSE value) than the full enhanced response surface approximation. As for the isotropic plate

Table 6.4: Strain distribution response surface approximation.

Model	Cp	R ²	Adj-R ²	%RMSE	%RMSE PRESS	%Avg Err	%RMSE	%Max Err
Stress	Data Set 1 (1,000 Data Points)				Data Set 2 (250 Data Points)			
Full Quadratic (78 terms)	78.000	0.9847	0.9834	-1.9226	-1.9910	-1.6692	-1.9128	-4.0613
Full Enhanced (98 terms)	98.000	0.9988	0.9986	-0.5510	-0.5794	-0.5543	-0.7094	-2.2777
Reduced Enhanced (60 terms)	40.549	0.9987	0.9987	-0.5510	-0.5622	-0.5876	-0.7404	-2.1497

example problem, the reduced response surface approximation is preferred due to its greater accuracy and smaller number of parameters, that represents a smoother response function.

Figure (6.11) shows the correlation between the finite element strain values and the strain values obtained from the response surface approximation. If there were 100% correlation between the finite element and the response surface approximation strain values, the actual data points indicated by the circular symbols would fall on the ideal diagonal line.

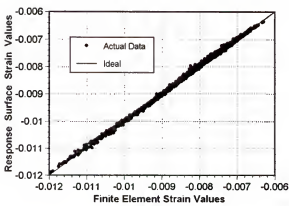


Figure 6.11: Response surface approximation compared to finite element strain values (Data Sets 1 and 2).

From Fig. (6.11) it is clear that the actual data points are close to the ideal diagonal line, indicating a high correlation or accuracy between the finite element and the response surface approximation strain values.

6.5.1.2. Stress distribution response surface approximations (Delamination failure)

The delamination failure load may be calculated from Eq. (6.6) and depends on the σ_{ZZ} , σ_{ZX} and σ_{ZY} inter-laminar stress components in the interfaces directly above and below the dropped plies. However, it was found that the contribution of the σ_{ZY} stress component was negligible for the current problem. Neglecting the contribution of the σ_{ZY} stress component influenced the average delamination failure index value for all of the data points of both Data Sets 1 and 2 by only 0.05%, while the maximum difference for all of these data points was only 0.6%. Response surface approximations were thus only constructed for the σ_{ZZ} and σ_{ZX} stress components, which may be written in non-dimensional function form (see Eq. 6.10) as:

$$\begin{aligned}\frac{\sigma_{ZZ}}{E_1} &= \frac{\sigma_{ZZ}}{E_1} \left(\frac{E_2}{E_1}, \frac{G_{12}}{E_1}, \frac{G_{23}}{E_1}, u_{12}, u_{23}, V_{1\_Drop}^*, V_{1\_Top}^*, V_{3\_Top}^*, V_{1\_Bottom}^*, V_{3\_Bottom}^*, \frac{x}{l} \right) \\ \frac{\sigma_{ZX}}{E_1} &= \frac{\sigma_{ZX}}{E_1} \left(\frac{E_2}{E_1}, \frac{G_{12}}{E_1}, \frac{G_{23}}{E_1}, u_{12}, u_{23}, V_{1\_Drop}^*, V_{1\_Top}^*, V_{3\_Top}^*, V_{1\_Bottom}^*, V_{3\_Bottom}^*, \frac{x}{l} \right) \quad (6.16)\end{aligned}$$

As was the case for the strain distribution response surface approximation, $V_{3\_Drop}^*$ depends linearly on $V_{1\_Drop}^*$ and is not required as a predictor variable (see Section 6.5).

For the stress distributions, the quadratic response surface approximations were enhanced by including all third order parameters associated with the four most important predictor variables, which were identified as E_2/E_1 , G_{12}/E_1 , $V_{1\_Drop}^*$ and x/l . Finally, the stepwise regression procedure and the Cp statistic were used to reduce the enhanced response

surface approximations. The predictive capabilities of all three response surface approximations, calculated from both Data Sets 1 and 2, are summarized in Table (6.5).

Table 6.5: Stress distributions (σ_{zz} and σ_{zx}) response surface approximations.

Model	Cp	R ²	Adj-R ²	%RMSE	%RMSE PRESS	%Avg Err	%RMSE	%Max Err
σ_{zz}	Data Set 1 (1,200 Data Points)				Data Set 2 (300 Data Points)			
Full Quad (78 terms)	78.000	0.9477	0.9441	21.3617	22.1134	17.7452	21.4668	66.8383
Full Enhanced (98 terms)	98.000	0.9893	0.9884	9.7205	10.1224	8.5347	11.1822	36.6367
Reduced Enhanced (46 terms)	14.057	0.9892	0.9887	9.5937	9.7773	8.2640	11.0264	36.9316
σ_{zx}	Data Set 1 (1,200 Data Points)				Data Set 2 (300 Data Points)			
Full Quad (78 terms)	78.000	0.9974	0.9973	-2.5458	-2.6324	-2.4617	-3.1395	-7.6673
Full Enhanced (98 terms)	98.000	0.9988	0.9987	-1.7696	-1.8395	-1.8273	-2.2827	-7.3187
Reduced Enhanced (46 terms)	13.942	0.9988	0.9987	-1.7465	-1.7784	-1.8275	-2.2718	-6.8726

As for the γ_{12} response surface approximations, the enhanced stress distribution response surface approximations are more accurate than the quadratic stress distribution response surface approximations. The reduced enhanced response surface approximations are slightly more accurate than the full enhanced response surface approximations (except for the %MaxErr value in the σ_{zz} response surface approximation and the %AvgErr value in the σ_{zx} response surface approximation). Again, the reduced response surface approximations are preferred due to its greater accuracy and smaller number of parameters that represent a

smoother approximate response function, and were used in the calculation of the possibility of failure. The predictive capabilities of the reduced response surface approximations are shown graphically in Fig. (6.12).

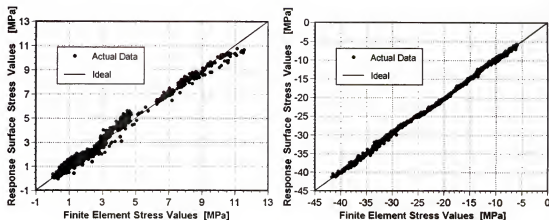


Figure 6.12: Response surface approximation compared to finite element stress values (Data Sets 1 and 2): (a) σ_{zz} distribution; (b) σ_{zx} distribution.

From Table (6.5) and Fig. (6.12) it is clear that the σ_{zz} stress distribution response surface approximation is not as accurate as the σ_{zx} stress distribution response surface approximation. However, the magnitude of σ_{zz} stress distribution is significantly smaller than the σ_{zx} stress distribution and the value of the failure index is thus dominated by σ_{zx} . The fact that the σ_{zx} stress distribution dominates the delamination failure index value is illustrated by calculating the failure index from the two reduced response surface approximations, with the results summarized in Table (6.6) and shown graphically in Fig. (6.13).

Table 6.6: Delamination failure index F_{Delam} (Data Set 2 with 300 data points).

%AvgErr	%RMSE	%MaxErr
1.4483	1.8173	5.4462

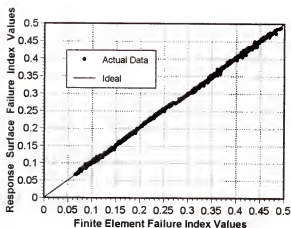


Figure 6.13: Delamination failure index F_{Delam} (calculated from the reduced σ_{zz} and σ_{zx} response surface approximations for Data Sets 1 and 2).

6.5.2. Possibility of Failure Response Surface Approximations

For the possibility of failure, a full cubic polynomial in the five possibility of failure predictor variables (56 parameters) was used as an initial response surface approximation. Recall that the five possibility of failure predictor variables are the weight (W) and the level of uncertainty associated with the elastic material properties (EL), the ply failure strength allowable (SP), the ply lay-up angle (PL) and the characteristic distance (CD). The D -Optimality criterion was used to select 200 data points for constructing the cubic possibility of failure response surface approximations. The D -Optimal data points were selected from a set of 7,776 candidate points that were created by considering a six level factorial design, consisting of six equally spaced levels in each of the design variables. Note that a six level factorial design is not required to generate candidate points for a cubic response surface approximation, but was used here since the small number of predictor variables allowed the use of six data points in each design variable for generating a comprehensive set of candidate points for selecting the D -Optimal data points.

Two hundred data points are a fairly large number of data points for constructing a polynomial with only 56 parameters. However, as in the isotropic example problem only data points with a possibility of failure not equal to either 0 or 1 were used in constructing the possibility of failure response surface approximations. Since it is not possible to know beforehand how many data points would be available for constructing the response surface approximations, a relatively large number of data points were thus selected. Two possibility of failure response surface approximations were constructed according to the two failure modes. The resulting predicted possibility of failure is then obtained from:

$$\hat{\Pi}_{(P-P_f)}(EL, SP, PL, CD, W) = \min(\hat{\Pi}_{PlyFail}(EL, SP, PL, CD, W), \hat{\Pi}_{DelamFail}(EL, SP, PL, CD, W)) \quad (6.17)$$

For the ply failure possibility of failure, 121 of the original 200 data points had a possibility not equal to either 0 or 1, while 197 of the data points had a delamination failure possibility of failure not equal to either 0 or 1. As before, the general response surface approximations were reduced using the mixed stepwise regression procedure and the C_p statistic, with the predictive capabilities of the possibility of failure response surface approximations summarized in Table (6.7).

As in the isotropic example problem, only a single data set was used to construct the possibility of failure response surface approximations and the $\%RMSE_{PRESS}$ value was used to compensate for the possibly overly optimistic error estimates.

Table 6.7: Possibility of failure response surface approximation.

Model	C _p	R ²	Adj-R ²	%RMSE	%RMSE PRESS
Ply Failure	Third Order Model (161 Data Points)				
Full (56 Terms)	56.000	0.9999	0.9997	0.9811	1.7165
Reduced (38 Terms)	29.783	0.9998	0.9998	0.9313	1.3871
Delamination Failure	Third Order Model (164 Data Points)				
Full (56 Terms)	56.000	0.9979	0.9970	3.1043	4.1788
Reduced (40 Terms)	30.982	0.9978	0.9972	3.0139	3.8650

CHAPTER 7

DROPPED PLY COMPOSITE LAMINATED PLATE OPTIMIZATION

The design for uncertainty problem considered here, involves minimizing a cost function of a typical aircraft structure, subject to uncertain problem parameters, for a specified allowable possibility of failure. A dropped ply composite laminated plate is considered as a typical structural component on such an aircraft. As was the case in the isotropic plate example problem, it is assumed that the structure will be built well into the future from materials not yet available. The cost function includes costs associated with increased structural weight, costs associated with material property characterization and costs associated with laying up the plies within a specified tolerance. The material characterization and manufacturing tolerance costs are applicable to the entire aircraft. The costs associated with increased weight are thus scaled to the entire aircraft, by assuming that the dropped ply composite laminated plate is representative of other structural components.

Fuzzy set theory is used to model the uncertainty using the limited data available to the designer. Additionally, response surface approximations are used throughout the design process, mainly to reduce the computational burden associated with designing for uncertainty, but also to simplify the integration of the analysis code with the optimization algorithm. Response surface approximations also filter out numerical noise in the approximate response function, thus allowing the use of a derivative-based optimization algorithm. The design optimization trades off costs associated with material characterization versus costs associated

with increased structural weight. The optimum results are presented in the form of design charts, trading off the cost against the allowable possibility of failure.

7.1. General Design Problem

A composite laminated plate with a change in thickness across its width, which is a result of the internal termination of plies (see Fig. 7.1), is the present design problem (more detail is presented in CHAPTER 6). It is assumed that the material properties and the applied load are uncertain, while manufacturing tolerances are modeled by considering uncertainty in the ply lay-up angles.

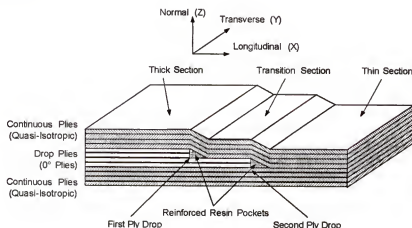


Figure 7.1: Two step dropped ply composite laminated plate.

Two failure mechanisms are considered during the design, delamination in the interfaces directly above and below the dropped plies, and ply failure. When considering both delamination and ply failure modes, the failure load P_f of the structure may be written as (see CHAPTER 6)

$$P_f = \text{Min} \left\{ \begin{array}{l} P_{Ply} \\ \frac{P_{Delam}}{1.5} \end{array} \right. \quad (7.1)$$

where:

$$P_{Ply} = \frac{P e_{12}}{\gamma_{12}} \quad P_{Delam} = \frac{P}{\sqrt{\left(\frac{\sigma_{zx}}{S_{13}}\right)^2 + \left(\frac{\sigma_{zz}}{Z_t}\right)^2}} \quad (7.2)$$

In Eqs. (7.1) and (7.2) P_{Ply} denotes the ply failure load, P_{Delam} the delamination failure load and P the applied load. Furthermore, γ_{12} and e_{12} are the shear strain and shear strain allowables, while σ_{zx} , σ_{zz} , S_{13} and Z_t are the inter-laminar stress components and stress allowables (material properties summarized subsequently in Table 7.1), respectively. Failure is then defined to occur when:

$$P - P_f \geq 0 \quad (7.3)$$

The factor of safety of 1.5 applied to the delamination failure load in Eq. (7.1) is used to account for the fact that the delamination failure load represents an ultimate load at which catastrophic failure occurs. In contrast, the ply failure load represents a limit load, indicating the onset of failure.

The objective of the design problem is to minimize a cost function (see Section 7.2), subject to an allowable possibility of failure. The results are presented in the form of design charts, trading of the cost against the allowable possibility of failure. The problem has five design variables, namely the weight (W) and the uncertainty associated with the elastic material properties (EL), the ply failure strength allowable (SP), the characteristic distance (CD), and the ply lay-up angle (PL). The uncertainty in the delamination strength allowables (SD) is also considered, however it is not a design variable, but is assumed to be twice the uncertainty associated with the ply failure strength allowable. A fixed uncertainty associated with the applied load of $\pm 20\%$ is assumed. The uncertain problem parameters and the nominal level of uncertainty associated with each parameter are summarized in Table (7.1).

Table 7.1: Uncertain problem parameters.

Variable	Nominal Values	Maximum Uncertainty	Design Variables
E_1	128,000 GPa	$\pm 10\%$	Elastic Properties (<i>EL</i>)
E_2, E_3	11,300 GPa	$\pm 10\%$	
G_{12}, G_{31}	6,000 GPa	$\pm 10\%$	
G_{23}	3 380 GPa	$\pm 10\%$	
ν_{12}, ν_{13}	0.3	$\pm 10\%$	
ν_{23}	0.35	$\pm 10\%$	Strength Properties (<i>SP</i> and <i>SD</i>)
Z_1	52.0 MPa	$\pm 40\%^\dagger$	
S_{13}	92.1 MPa	$\pm 40\%^\dagger$	
e_{12}	0.01552	$\pm 20\%$	Characteristic Distance (<i>CD</i>)
d_o	0.25 mm	$\pm 20\%$	
P	10 000 N	$\pm 20\%$	
Variable	Nominal Values	Maximum Uncertainty	Design Variables
θ_i	Default Lay-up of sub-laminates	$\pm 5^\circ$	Ply Lay-Up (<i>PL</i>)

† The uncertainty associated with the delamination strength allowables (*SD*) is assumed to be twice the uncertainty associated with the ply strength allowable (*SP*).

The uncertainty in the lay-up angle θ_i influences the five lamination parameters $V_{1\_Drop}^*$, $V_{1\_Top}^*$, $V_{3\_Top}^*$, $V_{1\_Bot}^*$ and $V_{3\_Bot}^*$ in the reduced stress and strain distribution response surface approximations used to calculate the failure load P_f . Assuming that $E_2=E_3$, $G_{12}=G_{31}$ and $\nu_{12}=\nu_{13}$ (as shown in Table 7.1), the possibility of failure due to the ply and delamination failure modes is calculated from a total of 16 uncertain problem parameters: E_1 , E_2 , G_{12} , G_{23} , ν_{12} , ν_{23} , Z_1 , S_{13} , e_{12} , $V_{1\_Drop}^*$, $V_{1\_Top}^*$, $V_{3\_Top}^*$, $V_{1\_Bot}^*$, $V_{3\_Bot}^*$, d_o and P .

7.2. Cost Function

The cost function used in the present dissertation includes costs associated with increased structural weight, costs associated with material property characterization and costs

associated with laying up the plies within a specified tolerance. The cost function may be written as

$$Cost = C_1 \text{ Weight} + C_2 \text{ Material} + C_3 \text{ PlyLayUp} \quad (7.4)$$

where the coefficients C_1 , C_2 and C_3 denotes the cost coefficients of the different components and will be discussed in detail in the rest of this section. For the design for uncertainty problem, the design variables are the five predictor variables (EL , SP , CD , PL , and W) of the possibility of failure response surface approximations.

In combining costs associated with weight, with costs associated with characterizing material properties and with manufacturing tolerances, a problem arises. The weight calculated is for a small structural component, while the material property characterization and manufacturing tolerance costs can be amortized over the entire aircraft, in fact, an entire fleet of aircraft made from the same materials. Here this problem is addressed by assuming that the dropped ply composite laminated plate is a representative structural component and its structural weight is thus scaled to an entire aircraft. This implicitly assumes that the number of aircraft that will be built from the same material is approximately equal to the number of material systems on one aircraft, which may exaggerate the material characterization costs.

The numbers used for the cost calculation in the present dissertation are intended to be only rough estimates, used here to demonstrate a methodology rather than performing a true cost calculation. A realistic cost calculation would require data from more structural components and more precise amortization of the cost of characterizing material properties or reducing manufacturing tolerances.

For the manufacturing and fuel costs associated with the structural weight, it was assumed that the failure load P_f is proportional to the representative composite material weight W of an entire aircraft as follows:

$$P_f \propto W \quad (7.5)$$

Additionally, if the representative composite material weight obtained from the baseline panel with no uncertainty is denoted by W_o and the corresponding failure load by P_{fo} , the failure load with uncertainty, P_f , may then be written as follows:

$$P_f = W \frac{P_{fo}}{W_o} \quad (7.6)$$

The baseline weight W_o of the structure is calculated by assuming that a typical aircraft has a structural composite material surface area of about 1,161 m² (12,500 ft²), corresponding to 180,000 of the 254.0 mm by 25.46 mm panels considered here. The weight of a single panel is calculated by multiplying the volume of the baseline panel V_{Panel} ($V_{Panel}=14,651\text{mm}^3$) by the density of the laminate ρ ($\rho=1,589 \text{ kg/m}^3$, Gibson⁹⁴ 1994, pp. 8 and 81). The baseline weight W_o is then obtained as follows:

$$W_o=180,000 (V_{Panel} \rho)=4,190 \text{ kg} \quad (7.7)$$

The cost coefficient for the weight component, C_t , is composed of the manufacturing and fuel cost per kilogram of structural weight, where the fuel cost is calculated over the entire lifetime of the aircraft. It was assumed that a kilogram of structural weight is worth \$550 in terms of material and manufacturing cost plus about 0.1 kilogram of fuel (or 2.94 cents) per flight. Assuming that the aircraft has a life span of 25 years and will complete 2,000 flights per year, one kilogram of structural weight results in \$1,470 of fuel costs over the life span of the aircraft. The total manufacturing and fuel cost component for structural weight is then:

$$C_1 \text{ Weight} = 2,020 W \quad (7.8)$$

The material characterization costs consist of several components, corresponding to the different material properties. It was assumed that the cost is inversely proportional to the level of uncertainty associated with the material properties as follows

$$C_2 \text{ Material} = C_2 \left(c_1 \frac{10}{EL} + c_2 \frac{20}{SP} + c_3 \frac{40}{SD} + c_4 \frac{20}{CD} \right) \quad (7.9)$$

where the c_i coefficients denote the relative cost of the different material properties and are summarized in Table (7.2). Percentage values are used to represent the uncertainty components EL , SP , SD and CD . Note that Eq. (7.9) is scaled to be equal to C_2 , when each uncertainty component is at its upper limit of uncertainty.

Table 7.2: Relative cost coefficients for the material characterization costs.

Property	Value
c_1	$\frac{1}{4}$
c_2	$\frac{1}{4}$
c_3	$\frac{1}{4}$
c_4	$\frac{1}{4}$

The overall cost coefficient C_2 was determined by assuming that \$2,000,000 were required to determine the material properties with uncertainty equal to the upper limit shown in Table (7.1). The final cost associated with the uncertainty in the material properties may thus be written as

$$C_2 \text{ Material} = 2,000,000 \left(\frac{10}{4 EL} + \frac{20}{4 SP} + \frac{40}{4 SD} + \frac{20}{4 CD} \right) \quad (7.10)$$

The cost of reducing the tolerance in the ply lay-up angles was again assumed to be inversely proportional to the level of uncertainty associated with the lay-up angles. For this

case, it was assumed that laying up the plies within a tolerance of 5° would correspond to a cost of \$50,000 and this cost component may then be written as

$$C_3 \text{ PlyLayUp} = 50,000 \frac{5}{PL} \quad (7.11)$$

where PL denotes the uncertainty in the ply lay-up angles measured in degrees.

7.3. Cost Optimization

In the design for uncertainty problem considered here, the cost function of Section 7.2 was minimized, subject to an allowable possibility of failure as a result of ply and delamination failure modes. The possibility of failure was obtained from the reduced possibility of failure response surface approximations of CHAPTER 6. The optimization was performed using the sequential linear programming subroutine provided with the commercial software package, Design Optimization Tools⁷⁰ (DOT) (1995). In order to avoid local minima, each optimization was repeated ten times from different starting points randomly distributed throughout the design space. No local minima were found. Note that the optimization algorithm used here is different from that used in the isotropic plate example problem (Microsoft Excel Version 8.0⁶⁹ 1997) and illustrates the fact that response surface approximations allow easy integration of the analysis code with any appropriate optimization algorithm. For the present example problem the Design Optimization Tools (DOT)⁷⁰ (1995) software was preferred since a large number of optimizations were required which could easily be performed from within a simple FORTRAN⁷¹ (1995) program.

7.3.1. Optimization Problem

The optimization problem for minimizing the cost function subject to a allowable possibility of failure, may be defined as:

Minimize:

$$Cost = 2,020 W + 2,000,000 \left(\frac{10}{4EL} + \frac{20}{4SP} + \frac{40}{4SD} + \frac{20}{4CD} \right) + 50,000 \left(\frac{5}{PL} \right) \quad (7.12)$$

Subject to:

$$\frac{\hat{\Pi}_{(P-P_f)}(EL, SP, PL, CD, W)}{\Pi_{allowable}} - 1 < 0 \quad (7.13)$$

Where $\hat{\Pi}_{(P-P_f)}$ denotes the approximate possibility of failure as obtained from the reduced response surface approximation of CHAPTER 6 and $\Pi_{allowable}$ denotes the allowable possibility of failure. The value of $\Pi_{allowable}$ was varied from 0 to 1 to study the influence of the allowable possibility of failure on the cost function and its individual cost components.

7.3.2. Results

First, the optimization was performed for a single $\Pi_{allowable}$ value equal to 0.2. This problem was solved to evaluate the methodology as well as the accuracy of the approximations, and the detailed results are summarized in Table (7.3).

For an allowable possibility of failure value equal to 0.2, the optimum cost function is equal to \$14.218 M. The total cost is dominated by the structural weight cost component (\$10.435 M or 73.4%). The structural weight is increased from the baseline value of 4,190 kg to 5,166 kg to increase the failure load of the structure, thus reducing the possibility of failure. Note that the optimizer limited the increase in the structural weight, by decreasing the uncertainty in the problem parameters, particularly the ply and delamination failure strength allowables. This decrease in the uncertainty associated with the problem parameters increased the material characterization and manufacturing tolerance costs from \$2.05 M to \$3.78 M. If the material characterization and design tolerance costs were not allowed to increase above

Table 7.3: Fuzzy set based optimum results for $\Pi_{allowable} = 0.2$.

Variable	Value	Cost
<i>EL</i>	7.18 %	\$0.6965 M
<i>SP</i>	9.73 %	\$1.0280 M
<i>SD</i>	19.5 %	\$1.0280 M
<i>CD</i>	14.5 %	\$0.6906 M
<i>PL</i>	0.73°	\$0.3402 M
<i>Load</i>	20.0 %	–
<i>Weight</i>	5,165.7 kg	\$10.435 M
<i>Cost</i>	–	\$14.218 M
Π_{ply} (Inactive)	0.0000 (0.0000)	–
Π_{Delam} (Active)	0.2001 (0.1856)	–

Values in parenthesis are predicted by possibility of failure response surface approximation.

\$2.05 M, the structural weight would have increased to 6,915 kg and the overall cost to \$16.02 M. The possibility of failure value calculated from the vertex method and the value predicted from the reduced possibility of failure response surface approximations (values in parenthesis) are both shown in Table (7.3), with the maximum difference equal to 7.2 %.

Next, a design chart of the cost compared to the allowable possibility of failure was generated. The allowable possibility of failure was varied from 0 to 1 with a constant interval of 0.05 and the results are shown graphically in Fig (7.2). From Fig. (7.2) it is clear that a decrease in the allowable possibility of failure results in an increase in the cost function. The relationship between the allowable possibility of failure and the cost function is almost linear with a change in the slope at an allowable possibility of failure equal to roughly 0.8. The practical application region of Fig. (7.2) is for an allowable possibility of failure between 0 and 0.4. Increasing the allowable possibility of failure above 0.4 would be impractical due to a high possibility of failure.

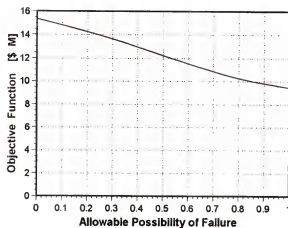


Figure 7.2: Cost compared to the allowable possibility of failure.

The change in the slope of the cost versus the allowable possibility of failure design chart at an allowable possibility of failure roughly equal to 0.8 corresponds to a change in the active failure mode, as shown in Fig. (7.3). The normalized constraint functions of Fig. (7.3) are inactive when negative, active when equal to zero and violated when positive.

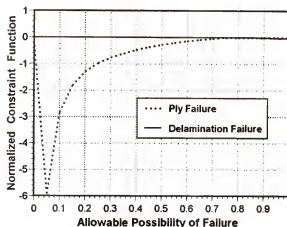


Figure 7.3: Normalized constraint functions compared to the allowable possibility of failure (active when normalized constraint function is equal to 0).

Finally, the cost associated with each of the design variables is shown as a function of the allowable possibility of failure in Fig. (7.4). As expected, the cost associated with the structural weight dominates the overall cost component. The cost associated with the

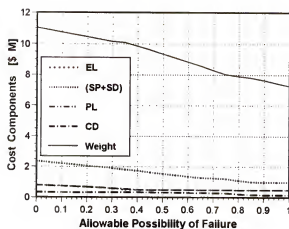


Figure 7.4: Individual cost components compared to the allowable possibility of failure.

strength allowables dominates the material characterization and manufacturing tolerance costs. Of all the uncertainties associated with the uncertain problem parameters, the uncertainty associated with the strength allowables changes the most with a change in the allowable possibility of failure. Reducing the uncertainty in the elastic material properties and the characteristic distance costs about the same, while reducing the uncertainty in the ply lay-up angles is fairly inexpensive. From Fig. (7.4) it is clear that the optimizer increases the structural weight with a decrease in the allowable possibility of failure in order to increase the failure load of the structure.

As noted previously, the material characterization costs can be amortized over many aircraft that are built from the same materials. In the present dissertation, it was implicitly assumed that the number of aircraft that will be built from the same materials is equal to the number of material systems for a single aircraft. Since this assumption may exaggerate the material characterization costs, a second set of optimizations was performed to demonstrate the cost calculation for the case where more aircraft will be built from the same materials. In this case it was assumed that the number of aircraft that will be built from the same materials

is equal to ten times the number of material systems for a single aircraft, thus reducing the material characterization costs by a factor of ten. The cost coefficient C_2 of Eq. (7.4) is thus reduced from 2,000,000 to 200,000 and the material characterization costs are thus defined as:

$$C_2 \text{ Material} = 200,000 \left(\frac{10}{4 EL} + \frac{20}{4 SP} + \frac{40}{4 SD} + \frac{20}{4 CD} \right) \quad (7.14)$$

Detailed results for the cost function with reduced material characterization costs (see Eq. 7.14) are summarized in Table (7.4) for the case where the allowable possibility of failure is equal to the 0.2. As expected, the reduced material characterization costs resulted in a reduction in the level of uncertainty associated with the material properties. This reduction in the level of uncertainty associated with the material properties, resulted in a decrease in both the structural weight cost component and the manufacturing tolerance cost component. The costs associated with the structural weight were decreased from \$10.435 M to \$8.798 M, the costs associated with material characterization were reduced from \$3.443 M to \$0.879 M,

Table 7.4: Fuzzy set based optimum results for $\Pi_{\text{allowable}} = 0.2$.

Variable	Value	Cost
<i>EL</i>	2.80 %	\$0.1783 M
<i>SP</i>	3.85 %	\$0.2597 M
<i>SD</i>	7.70 %	\$0.2597 M
<i>CD</i>	5.51 %	\$0.1815 M
<i>PL</i>	0.91°	\$0.2760 M
<i>Load</i>	20.0 %	–
<i>Weight</i>	4,355.3 kg	\$8.7976 M
<i>Cost</i>	–	\$9.9530 M
Π_{ply}	0.2000	
(Active)	(0.1962)	–
Π_{Delam}	0.2000	
(Active)	(0.1818)	–

Values in parenthesis are predicted by possibility of failure response surface approximation.

and the costs associated with the manufacturing tolerances were reduced from \$0.340 M to \$0.276 M. The overall cost was reduced from \$14.218 M to \$9.953 M. The greatest reduction in uncertainty was observed for the uncertainty associated with the strength allowables and the characteristic distance. The reduction in the uncertainty associated with the material properties resulted in a slight increase in the tolerance of the ply lay-up angle and a reduction in the structural weight. The tolerance in the ply lay-up angle was increased from 0.73° to 0.91° and the structural weight was reduced from 5,166 kg to 4,355 kg. The possibility of failure value calculated from the vertex method and the value predicted from the reduced possibility of failure response surface approximations (values in parenthesis) are both shown in Table (7.4). For ply failure, the values of the possibility of failure obtained from the vertex method and the reduced response surface approximation respectively differs by 1.9%, while the difference is equal to 9.1% for delamination failure.

The cost function is shown graphically as a function of the allowable possibility of failure in Fig. (7.5). The allowable possibility of failure was varied from 0 to 1, with a constant interval of 0.05. As before, a decrease in the allowable possibility of failure results in

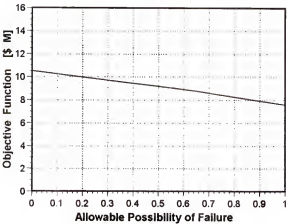


Figure 7.5: Cost compared to the allowable possibility of failure.

an increase in the cost function, with an almost linear relationship between the allowable possibility of failure and the cost function. The maximum value of the cost function is, however, reduced from roughly \$15.3 M to \$10.5 M.

The normalized constraint functions are shown graphically in Fig. (7.6) as a function of the allowable possibility of failure. In this case, both constraints are active over a wide range of allowable possibility of failure values. However, for small values of the allowable possibility of failure, only the delamination failure constraint is active, while only the ply failure constraint is active for large values of the allowable possibility of failure.

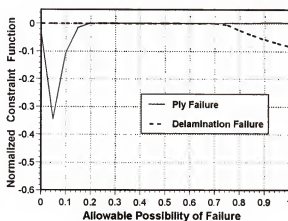


Figure 7.6: Normalized constraint functions compared to the allowable possibility of failure (active when normalized constraint function is equal to 0).

Finally, the individual cost components of the objective function (the cost function) are shown graphically in Fig. (7.7) as a function of the allowable possibility of failure. The individual cost components are represented by curves that are similar to those of the initial example problem. As expected, the absolute values of the material characterization costs are much lower, while the structural weight cost component is even more dominant in terms of the overall cost. Note, however, that the absolute value of the structural weight cost

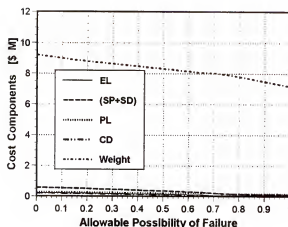


Figure 7.7: Individual cost components compared to the allowable possibility of failure.

component is lower than in the initial problem, due to more accurate material property characterization.

7.4. Reduction in Computational Cost

The reduction in computational cost is mainly associated with the response surface approximations that were constructed for the stress and strain distributions. These response surface approximations of CHAPTER 6 were used to replace computationally expensive finite element analyses during the evaluation of the possibility of failure. Compared to the isotropic plate example problem, this process resulted in even more substantial reductions in computational cost.

The reductions in computational cost are illustrated for a single two level optimization problem. The evaluation of the possibility of failure for a single α level cut requires $2^{10} \times 20 \times 20 + 2^{11} \times 20 \times 20 = 1,228,800$ finite element analyses when no response surface approximations are used. Recall that there are a total of 16 uncertain problem parameters, 14 associated with the ply failure mode and 15 associated with the delamination failure mode. Additionally, it was necessary to consider all boundary points (20 data points) for each of the

$V_{1\_Top}^*$ and $V_{3\_Top}^*$ as well as the $V_{1\_Bot}^*$ and $V_{3\_Bot}^*$ design spaces, respectively (see CHAPTER 6).

When not using response surface approximations, an estimate of the required number of finite element analyses to perform a single optimization is obtained from the product of the following four numbers:

Average number of design optimization iterations:	10
Average number of $\Pi_{(P,P_f)}$ evaluations per iteration:	10
Average number of α level cut evaluations per $\Pi_{(P,P_f)}$ evaluation:	5
Number of finite element analyses per α level cut evaluation of $\Pi_{(P,P_f)}$:	1,228,800
Total number of finite element analyses required per optimization:	<u><u>614,400,000</u></u>

In contrast, the stress distribution and buckling load response surface approximations were constructed, and their predictive capabilities evaluated, from a total of only 250 finite element analyses. These observations indicates that response surface approximations is an effective approach for reducing the computational cost associated with performing fuzzy set based designing for uncertainty.

As in the isotropic plate example problem, the use of response surface approximations allowed multiple optimizations to be conducted at little additional computational cost. It was thus possible to construct and investigate design charts of the optimum results, based on multiple optimizations.

CHAPTER 8

CONCLUDING REMARKS

The use of response surface approximations in expensive structural optimization problems was investigated in this dissertation. Designing for uncertainty of a stepped plate was considered as a representative example of an expensive structural optimization problem. Both an isotropic and a composite laminated structure were considered. For the composite laminated structure, the change in thickness is a result of the internal termination of plies. It was assumed that the structures would be built well into the future, from materials not yet available. Little information is thus available on the uncertainty, and thus fuzzy set theory was used to model the uncertainty in the material properties, the geometry and the applied load. Designing for uncertainty, where the uncertainty is modeled by fuzzy set theory, is a good example of an expensive structural optimization problem, since it is a two level optimization problem.

When using an approximate response function in optimization, the accuracy of the response function is of the utmost importance, since optimization procedures have the general tendency of exploiting weaknesses in the response function formulation. In the present dissertation, highly accurate response surface approximations were obtained by using a variety of mathematical and statistical tools. Dimensional analysis was used to identify the intrinsic variables of the response functions. Using these variables to construct the response surface approximations improved the accuracy while reducing the number of predictor variables of the resulting response surface approximations. Additionally, higher order (cubic and quartic

instead of the more traditional quadratic) polynomials were used as initial response models, which resulted in more accurate response surface approximations that were valid over the entire design space. Mixed stepwise regression and the C_p statistic were used to eliminate insignificant parameters from these initial models, greatly reducing the number of parameters in the final response surface approximations. For the composite laminated plate, where it was not feasible to evaluate a large number of data points for constructing the response surface approximations, statistical design of experiments in the form of the D -Optimality criterion was used to find a small set of data points while maintaining accuracy. Finally, a detailed error analysis was performed to ensure that the response surface approximations were indeed of high quality.

Two sets of response surface approximations were constructed during the design process. The first set was used to calculate the possibility of failure, which is an expensive global optimization problem. In this case, the response surface approximations were mainly used to reduce the computational burden associated with calculating the possibility of failure. The second set of response surface approximations was constructed to represent the possibility of failure in terms of the overall design variables of the problem and was mainly used to simplify the integration of the analysis code and the optimization algorithm. Overall, the response surface approximations resulted in substantial reductions in computational cost since:

1. It was cheaper to construct and to evaluate the predictive capabilities of the approximations than to perform a single two level optimization problem.
2. The computational cost was shifted from the optimization problem to that of constructing the approximations, thus allowing multiple optimizations to be conducted at little additional computational cost.

The reduction in computation cost, especially shifting the cost from the optimization problem to that of constructing the approximations, allowed the construction of weight versus cost design charts based on the results obtained from multiple optimizations. These design charts provide a valuable tool during the initial design stages. In the present dissertation, these weight versus cost design charts would have been very costly to construct without the use of response surface approximations.

The main problem associated with the use of response surface approximations in practical problems is that the cost of constructing the approximations is subject to the curse of dimensionality. The curse of dimensionality implies that the number of data points required to construct a response surface approximation is greatly increased with an increase in the order of the response surface approximation and/or the number of design variables. Response surface approximations are thus not applicable to all problems, and the practical application of response surface approximations is limited to problems with less than 25 to 30 design variables.

In general, it has been illustrated that response surface approximations provide an effective approach for solving expensive structural optimization problems with a relatively small number of design variables. Response surface approximations resulted in substantial reductions in computational cost, greatly simplified the integration of the analysis codes and optimization algorithms, and allowed the construction of weight versus cost design charts.

APPENDIX A: DROPPED PLY COMPOSITE LAMINATED PLATE PROBLEM EVOLUTION

A.1. Introduction

The internal termination of plies (referred to as dropped plies) is an effective method for reducing the thickness of composite laminated structures. Unfortunately, the dropped plies result in severe stress concentrations due to the stiffness and thickness discontinuities associated with the internal termination of the plies. These stress concentrations may greatly reduce the strength of the structure, thus offsetting the advantages of reducing the thickness by terminating plies. Various analytical, numerical and experimental studies (Refs. 79 to 90) indicated that delamination is a common mode of failure for dropped ply composite laminated structures.

The final transition zone configuration of the dropped ply composite laminated plate considered in this dissertation, was obtained by performing a numerical investigation using detailed finite element analyses. Additionally, manufacturing constraints were also considered. The goal of the numerical investigation was to study the influence of the transition zone configuration on the strength of the dropped ply composite laminated plate. During the numerical investigation, a problem similar to that studied by Curry, Johnson and Starnes⁸² (1992) and Dávila and Johnson⁸⁷ (1993) was used as a baseline problem. This problem is discussed in detail in CHAPTER 6 and is shown schematically in Fig. A.1. For the baseline problem, four 0° plies were dropped with quasi-isotropic sub-laminates above and below the dropped plies.

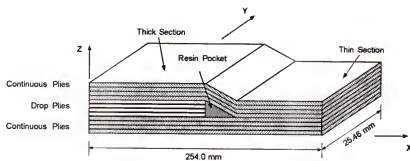


Figure A.1: Baseline dropped ply composite laminated plate.

A.2. Numerical Investigation

Based on the experimental and numerical work of Curry, Johnson and Starnes⁸² (1992) and the numerical work of Dávila and Johnson⁸⁷ (1993), the numerical investigation presented here considered delamination failure that occurs in the interfaces directly above and below the dropped plies as the critical failure mode. A quadratic failure index was used to predict delamination failure initiation (see CHAPTER 6 for more detail), which may be summarized as follows.

$$F_{Delam}^2 = \left(\frac{\sigma_{zx}}{S_{13}} \right)^2 + \left(\frac{\sigma_{zy}}{S_{23}} \right)^2 + \left(\frac{\sigma_{zz}}{Z_i} \right)^2 \quad \text{where } \sigma_{zz} > 0 \quad (\text{A.1})$$

This failure index is evaluated at a characteristic distance away from the ply drop. Based on the results of Dávila and Johnson⁸⁷ (1993), a characteristic distance of 0.25 mm was assumed as a characteristic distance throughout this dissertation.

The influence of the drop zone length (shown in Fig. A.2) on the delamination strength of the dropped ply composite laminated plate was investigated first. The drop zone length was varied between 0.0887 mm and 2.850 mm, changing the drop zone angle (shown in Fig. A.2) from 10° to 80°.

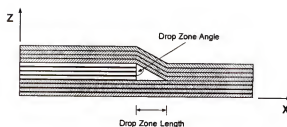


Figure A.2: Transition zone parameters for the baseline dropped ply composite laminated plate.

The delamination failure index compared to the drop zone angle is shown graphically in Fig. (A.3), where a smaller drop zone angle corresponds to a shorter drop zone length.

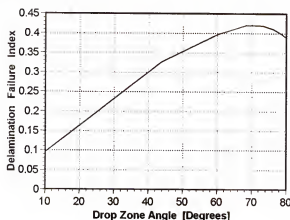


Figure A.3: Delamination failure index compared to the drop zone angle for the baseline dropped ply composite laminated plate.

Figure (A.3) indicates that an increase in the drop zone length, corresponding to an increase in the drop zone angle, results in a decrease in the delamination strength of the composite laminated plate. The delamination failure index is thus dominated by the severity of the stiffness discontinuity that occurs at the ply drop, rather than the severity of the thickness discontinuity which becomes less severe for a longer drop zone length.

The results of Fig. (A.3) were validated by studying the accuracy of the finite element model and the validity of using equivalent orthotropic material properties for each sub-

laminate. The accuracy of the finite element model was investigated by refining the finite element mesh in order to evaluate the convergence of the model. The validity of the equivalent orthotropic material properties assumption was investigated by modeling each ply of each sub-laminate as an individual orthotropic layer, orientated at a prescribed angle with respect to the x-axis. The results obtained from these investigations indicated that: (1) the original finite element model was not converged; and (2) the use of equivalent orthotropic material properties have an insignificant influence on the delamination failure index. Based on these results, the finite element model was refined and the refined model was used throughout the rest of this dissertation (see CHAPTER 6 for more detail about the refined model). However, after the finite element model was refined, the delamination strength of the dropped ply composite laminated plate still increased with a decrease in the drop zone length.

Based on the validation of the results shown in Fig. (A.3), the following hypothesis has been formulated to explain the results:

The delamination failure index value of a dropped ply composite laminated plate depends on the severity of the change in stiffness between the thick and the thin sections of the composite laminated plate.

This hypothesis implies that, when a resin pocket is present, a shorter drop zone length results in a lower delamination failure index, since the severity of the stiffness discontinuity is decreased. The resin pocket has a much lower stiffness than the surrounding sub-laminates and a longer drop zone length increases the compliance of the resin pocket. The severity of the stiffness discontinuity is thus increased with an increase in the drop zone length.

The hypothesis was evaluated by considering a number of alternative transition zone configurations, thus changing the stiffness of the transition zone and the severity of the stiffness discontinuity. The stiffness of the transition zone was increased by reducing the size of the resin pocket as well as by reinforcing the resin pocket with 90° fibers, measured relative to the x -axis. The size of the resin pocket was reduced by dropping the 0° plies in two steps, which resulted in two smaller resin pockets, and by tapering the dropped plies in the drop zone, thus completely eliminating the resin pocket (see Fig. A.4).

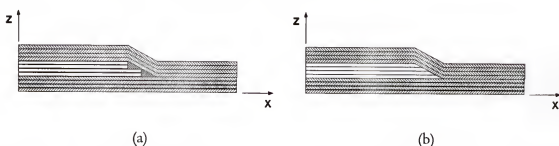


Figure A.4: Modified transition zone configurations: (a) two step configuration; (b) tapered dropped ply configuration.

The delamination failure index for both a short drop zone length (i.e., 0.0887 mm or 10° drop zone angle) and a long drop zone length (i.e., 1.273 mm or 68.4° drop zone angle) are shown graphically in Fig. (A.5) for the baseline, the two step, and the tapered configurations. For the baseline and the two step configurations, two cases were considered. In the first case, the resin pocket consisted of neat resin, and in the second case the resin pocket was reinforced with 90° fibers.

From Fig. (A.5) it is clear that as the stiffness of the transition zone is increased, the delamination failure index decreases. When the resin pocket is completely eliminated (tapered configuration) the results of Fig. (A.3) are reversed, since a longer transition zone increases the delamination strength of the structure. Additionally, the tapered configuration resulted in the lowest value of the delamination failure index, corresponding to a significant increase in the

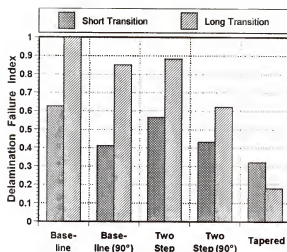


Figure A.5: Delamination failure index for different transition zone configurations.

delamination strength of the structure. Unfortunately, high manufacturing costs limit the usefulness of the tapered configuration. The two step configuration with 90° fibers reinforcing the resin pockets is the best practical configuration. For all cases, except the tapered configuration, it was found that a short transition zone resulted in the lowest value of the delamination failure index. However, a shorter transition zone increases the severity of the thickness discontinuity. Since a shorter transition zone results in lower values of the delamination failure index, even though the severity of the thickness discontinuity is increased, it may be concluded that the thickness discontinuity has a small influence on the delamination failure index value.

Based on the fact that a short transition zone was the practical configuration with the lowest delamination failure index, it was decided to investigate ply failure as a second failure mechanism. Ply failure occurs due to high in-plane stresses resulting from the thickness discontinuity, but is also influenced by the internal stiffness discontinuity. Using the maximum strain failure criterion, the topmost ply of the thinnest section of the composite

laminated plate was identified as the most critical ply (see CHAPTER 6 for more detail). In all cases the in-plane shear strain was the most critical strain component, with a singularity occurring at the re-entrant corner of the composite laminated plate. Based on the maximum strain failure criterion, the ply failure index is defined as:

$$F_{py} = \frac{\gamma_{12}}{e_{12}} \quad (\text{A.2})$$

As was the case for the delamination failure index of Eq. (A.1), the ply failure index is also calculated at a characteristic distance away from the singularity, in this case the re-entrant corner. Throughout this dissertation, the same characteristic distance (i.e., 0.25 mm) was assumed for both the delamination and ply failure modes. The ply failure index value is shown graphically in Fig. (A.6), for the five transition zone configurations shown in Fig. (A.5).

From Fig. (A.6) it is observed that increasing the stiffness of the transition zone has a much smaller influence on the ply failure index value as compared to the influence on the

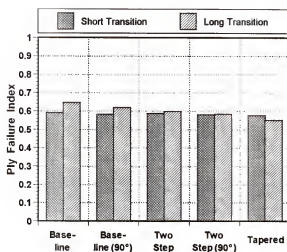


Figure A.6: Ply failure index for different transition zone configurations.

delamination failure index value (see Fig. A.5). The increased stiffness of the transition zone has almost no influence on the ply failure index value for the short drop zone length cases, but does slightly improve the ply failure index value for the long drop zone length cases. As was the case for the delamination failure index, a short drop zone length is the practical configuration with the lowest ply failure index value (see Fig. A.6). For all the practical configurations, the severity of the stiffness discontinuity dominates the ply failure index value, since the severity of the thickness discontinuity increases with a decrease in the drop zone length, while the severity of the stiffness discontinuity decreases with a decrease in the drop zone length.

Figure (A.7) shows the critical failure indices when considering both delamination and ply failure, for the five cases shown in Figs. (A.5) and (A.6). In order to make a direct comparison between the delamination and the ply failure indices, it is important to remember that the delamination failure load represents an ultimate load condition, at which catastrophic failure occurs. In contrast, the ply failure load represents a limit load condition that indicates the onset of failure. Within the aircraft industry the ultimate load is typically assumed to be 1.5 times larger than the limit load. This ratio of 1.5 was used as a factor of safety in evaluating the delamination failure criterion, while a factor of safety of 1.0 was used in evaluating the ply failure criterion. Note that the factor of safety of 1.5 was incorporated in the results of Figs. (A.5) and (A.7), but not in the initial results of Fig. (A.3).

In Fig. (A.7) the ply failure mode is critical for all short drop zone length cases (except for the baseline configuration), while the delamination failure mode is critical for all long drop zone length cases (except for the tapered configuration). Additionally, the two step configuration, where the resin pockets are reinforced with 90° fibers, emerged as the best

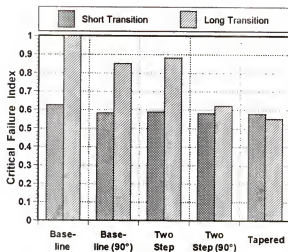


Figure A.7: Critical failure index for different transition zone configurations.

practical configuration. Figure (A.7) indicates that for the two step configuration with reinforced resin pockets, a short transition zone results in the lowest critical failure index value. However, the difference in the failure indices of the short and long drop zone lengths is small and, due to manufacturing constraints, it was decided to use a drop zone length equal to eight ply thicknesses (i.e., 1.0056 mm or 63.4° drop zone angle).

This configuration was further investigated, by considering the influence of separating the two ply drops on the strength of the composite laminated plate. Figure (A.8) shows the separation in the two ply drops schematically. For this part of the investigation, the separation distance was varied between 0 mm and 4.5 mm (i.e., between 0 ply thicknesses and 35 ply thicknesses) and the results are shown graphically in Fig. (A.9).

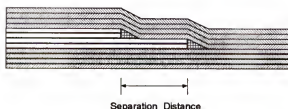


Figure A.8: Two step configuration with separated ply drops.

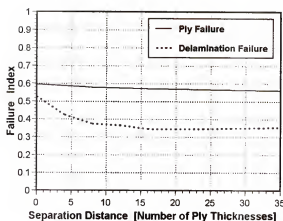


Figure A.9: Failure index compared to the separation distance for the two step configuration with reinforced resin pockets.

Figure (A.9) indicates that the separation distance has a much higher influence on the delamination failure index value than on the ply failure index value. For the delamination failure index, there is a 32.8% change in the failure index values when changing the separation distance from 0 to 35 ply thicknesses, while there is only a 5.8% change in the values of the ply failure index. Increasing the separation distance above roughly 15 ply thicknesses has very little influence on the failure indices, since the two stress singularities corresponding to the two ply drops become separated and do not influence each other. However, due to manufacturing constraints, a separation distance of 20 ply thicknesses was used in the final configuration considered as an example problem in this dissertation. This final two step dropped ply composite laminated plate configuration, where the ply drops are separated by 20 ply thicknesses and the resin pockets are reinforced with 90° fibers, is shown schematically in Fig. (A.10).

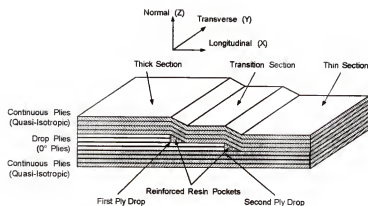


Figure A.10: Two step configuration (ply drops separated by 20 ply thicknesses) with reinforced resin pockets.

APPENDIX B: DIMENSIONAL ANALYSIS

B.1. Introduction

Dimensional analysis is based on the simple idea that any physical law is independent of the arbitrary chosen units of measurement used for describing a specific system. Instead, any physical law possesses a fundamental property that may be expressed in terms of variables intrinsic to the system. Dimensional analysis is an important tool in developing approximate response functions based on experimental data (either physical or numerical experiments). Dimensional analysis may greatly reduce the number of experiments required to construct the approximate response function, by reducing the number of variables that describe the response. The variables that describe the response are known as governing variables (in this dissertation these variables are also referred to as predictor variables). Dimensional analysis is discussed in detail in Barenblatt⁴⁸ (1996, pp.1-63), which makes use of the Buckingham Π -Theorem to formulate a non-dimensional response quantity in terms of a group of non-dimensional variables. The number of non-dimensional variables is in general smaller than the original number of governing variables. The Buckingham Π -Theorem (Buckingham⁴⁹ 1915) may be stated as:

A physical relationship between some dimensional (generally speaking) quantity and several dimensional governing variables can be rewritten as a relationship between some dimensionless quantity and several dimensionless products of the governing variables; the number of dimensionless products is equal to the total

number of governing variables minus the number of governing variables with independent dimensions

An independent dimension is defined as a dimension that cannot be expressed in terms of a product of powers of the remaining variables.

B.2. Application of the Buckingham Π -Theorem

In constructing an approximate response function of a system, the first step is to express the response in terms of the governing variables (predictor variables) as follows:

$$y = f(z_1, z_2, \dots, z_k) \quad (\text{B.1})$$

The governing variables may then be divided into two groups such that the m governing variables z_1 to z_m have independent dimensions, while the dimensions of the response y and remaining $(k - m)$ governing variables may be expressed in terms of the m governing variables with independent dimensions, as follows:

$$y = f(z_1, \dots, z_m, z_{m+1}, \dots, z_k) \quad (\text{B.2})$$

$$[y] = [z_1]^{a(1)} [z_2]^{a(2)} \dots [z_m]^{a(m)}$$

$$[z_{m+j}] = [z_1]^{a(m+j)(1)} [z_2]^{a(m+j)(2)} \dots [z_m]^{a(m+j)(m)} \quad (\text{B.3})$$

Note that $j \in [1, (k - m)]$ and that the independent dimensions account for all the fundamental dimensional units of the problem, typically force [F], length [L] and possibly time [T] for a mechanics problem.

Based on Eq. (B.3) it is thus possible to define a non-dimensional response quantity Π and a set of $(k - m)$ non-dimensional variables Π_i as follows:

$$\Pi = \frac{y}{z_1^{a(1)} z_2^{a(2)} \dots z_m^{a(m)}}$$

$$\Pi_j = \frac{z_{m+j}}{z_1^{a(m+j)(1)} z_2^{a(m+j)(2)} \dots z_m^{a(m+j)(m)}} \quad (\text{B.4})$$

According to the Buckingham Π -Theorem the non-dimensional response quantity is only dependent on the $(k - m)$ non-dimensional variables as follows:

$$\Pi = F(\Pi_1, \Pi_2, \dots, \Pi_{(k-m)}) \quad (\text{B.5})$$

B.3. Example Problem

As an example problem consider a cantilevered beam with length L and circular cross section with diameter D as shown in Fig (B.1).

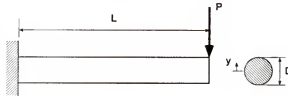


Figure B.1: Cantilevered beam example problem.

The bending stress σ_B at the root of the cantilevered beam may then be written in terms of the governing variables (predictor variables) as:

$$\sigma_B = f(P, L, D, y) \quad (\text{B.6})$$

where P is the applied load and y is the distance, measured from the neutral axis (see Fig. B.1), at which the bending stress σ_B is evaluated. The fundamental dimensional units of the problem are force [F] and length [L]. The applied load P (dimension [F]) and diameter D (dimension [L]) are selected as the governing variables with independent dimensions. The

non-dimensional bending stress σ_b may then be written in terms of two non-dimensional variables as follows:

$$\frac{\sigma_b D^3}{P} = F\left(\frac{L}{D}, \frac{y}{D}\right) \quad (\text{B.7})$$

The dimensional analysis thus reduced the number of predictor variables from four to only two. In terms of a quadratic response surface approximation the reduction in predictor variables would reduce the number of parameters in the full response surface approximation from 15 to only 6, and reduce the number of parameters in a quartic response surface approximation from 35 to only 10.

REFERENCES

- [1] Kaufman, M., Balabanov, V., Grossman, B., Mason, W. H., Watson, L. T., and Haftka, R. T., "Multidisciplinary Optimization via Response Surface Techniques," Proceedings of the 36th Israel Conference on Aerospace Sciences, Israel, February 21-22, 1996, pp. A-57-A-67.
- [2] Burden, R. L., and Faires, J. D., *Numerical Analysis, Fourth Edition*. Boston: PWS-Kent Publishing Company, 1989.
- [3] Giunta, A. A., Dudley, J. M., Narducci, R., Grossman, B., Haftka, R. T., Mason, W. H., and Watson, L. T., "Noisy Aerodynamic Response and Smooth Approximations in HSCT Design," Proceedings of the 5th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Panama City, Florida, September 7-9, 1994, Paper No. 94-4376, pp. 1117-1128.
- [4] Dudley, J., Huang, X., MacMillin, P. E., Grossman, B., Haftka, R. T., and Mason, W. H., "Multidisciplinary Optimization of the High-Speed Civil Transport," 34th Aerospace Sciences Meeting and Exhibit, Reno, Nevada, January, 1995, Paper No. 95-0125.
- [5] Haftka, R. T., Scott, E. P., and Cruz, J. R., "Optimization and Experiments - A Survey," *Applied Mechanics Reviews*, July, 1998, Vol. 51, No. 7, pp. 435-448.
- [6] Myers, R. H., and Montgomery, D. C., *Response Surface Methodology: Process and Product Optimization Using Designed Experiments*. New York: John Wiley & Sons, Inc., 1995.
- [7] Box, G. E. P., and Draper, N. R., *Empirical Model-Building and Response Surfaces*. New York: John Wiley and Sons, Inc., 1987.
- [8] Box, G. E. P., and Wilson, K. B., "On the Experimental Attainment of Optimum Conditions," *Journal of the Royal Statistical Society*, 1951, Series B, pp. 1-45.
- [9] Box, G. E. P., and Behnken, D. W., "Some New Three-Level Designs for the Study of Quantitative Variables," *Technometrics* 30, 1960, pp. 1-40.
- [10] Balabanov, V., Kaufman, M., Knill, D. L., Haim, D., Golovidov, O., Giunta, A. A., Grossman, B., Mason, W. H., Watson, L. T., and Haftka, R. T., "Dependence of Optimal Structural Weight on Aerodynamic Shape for a High Speed Civil Transport," Proceedings of the 6th AIAA/NASA/ISSMO Symposium on Multidisciplinary

Analysis and Optimization, Bellevue, Washington, September 4-6, 1996, Paper No. 96-4046, Part 1, pp. 599-612.

- [11] Venter, G., and Haftka R. T., "Minimum-Bias Based Experimental Design for Constructing Response Surfaces in Structural Optimization," Proceedings of the 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Kissimmee, Florida, April 7-10, 1997, Paper No. 97-1053, Part 2, pp. 1225-1238.
- [12] Harrison, P. N., Le Riche, R., and Haftka, R. T., "Design of Stiffened Composite Panels by Genetic Algorithm and Response Surface Approximations," Proceedings of the 36th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, New Orleans, Louisiana, April 10-13, 1995, Paper No. 95-1163, Part 1, pp. 58-65.
- [13] Sellar, R. S., and Batill, S. M., "Concurrent Subspace Optimization Using Gradient-Enhanced Neural Network Approximations," Proceedings of the 6th AIAA/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Bellevue, Washington, September 4-6, 1996, Paper No. 96-4019, Part 1, pp. 319-330.
- [14] Wujek, B. A., Renaud, J. E., Batill, S. M., and Brockman, J. B., "Concurrent Subspace Optimization Using Design Variable Sharing in a Distributed Computing Environment," Concurrent Engineering: Research and Applications, December, 1996, Vol. 4, No. 4, pp. 361-377.
- [15] Rodríguez, J. F., Renaud, J. E., and Watson, L. T., "Convergence of Trust Region Augmented Lagrangian Methods using Variable Fidelity Approximation Data," Presented at the Second World Congress of Structural and Multidisciplinary Optimization, Zakopane, Poland, May 26-30, 1997.
- [16] Cramer, E. J., and Carter, M. L., "Global Optimization of Multi-Element Airfoil," Proceedings of the 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Kissimmee, Florida, April 7-10, 1997, Paper No. 97-1330, Part 2, pp. 1377-1382.
- [17] Barthelemy, J.-F. M., and Haftka, R. T., "Approximation Concepts for Optimum Structural Design - A Review," Structural Optimization, 1993, Vol. 5, No. 3, pp. 129-144.
- [18] Haftka, R. T., and Gürdal, Z., *Elements of Structural Optimization. Third revised and expanded edition.* The Netherlands: Kluwer Academic Publishers, 1993.
- [19] Schmit, L. A., and Farshi, B., "Some Approximation Concepts for Structural Synthesis," AIAA Journal, 1974, Vol. 12, No. 5, pp. 692-699.
- [20] Schmit, L. A., and Miura, H., "Approximation Concepts for Efficient Structural Synthesis," NASA CR-2552, March, 1976.

- [21] Haftka, R. T., Nachlas, J. A., Watson, L. T., and Desai, R., "Two-Point Constraint Approximation in Structural Optimization," *Computer Methods in Applied Mechanics and Engineering*, 1989, Vol. 60, pp. 289-301.
- [22] Fadel, G. M., Riley, M. F., and Barthelemy, J.-F. M., "Two Point Exponential Approximation Method for Structural Optimization," *Structural Optimization*, 1990, Vol. 2, pp. 117-124.
- [23] Snyman, J. A., and Stander, N., "New Successive Approximation Method for Optimum Structural Design," *AIAA Journal*, June, 1994, Vol. 32, No. 6, pp. 1310-1315.
- [24] Wang, L., and Grandhi, R. V., "Efficient Safety Index Calculation for Structural Reliability Analysis," *Computers and Structures*, 1994, Vol. 52, No. 1, pp. 103-111.
- [25] Wang, L., and Grandhi, R. V., "Improved Two-Point Function Approximations for Design Optimization," *AIAA Journal*, 1995, Vol. 33, No. 9, pp. 1720-1727.
- [26] Wang, L., Grandhi, R. V., and Hopkins, D. A., "Structural Reliability Optimization Using an Efficient Safety Index Calculation Procedure," *Proceedings of the 35th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Hilton Head, South Carolina, April 18-20, 1994, Paper No. 94-1416, Part 2, pp. 853-863.
- [27] Wang, L., Grandhi, R. V., and Canfield, R. A., "Multi-Point Constraint Approximations in Structural Optimization," *Proceedings of the 5th AIAA/ASAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Panama City Beach, Florida, September 7-9, 1994, Paper No. 94-4280, Part 2, pp. 269-280.
- [28] Wang, L., and Grandhi, R. V., "Multi-Point Approximations: Comparisons using Structural Size, Configuration and Shape Design," *Proceedings of the 37th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Salt Lake City, Utah, April 15-17, 1996, Paper No. 96-1384, Part 1, pp. 594-603.
- [29] Rodríguez, J. F., Renaud, J. E., and Watson, L. T., "Trust Region Augmented Lagrangian Methods for Sequential Response Surface Approximation and Optimization," *Proceedings of the 1997 ASME Design Engineering Technical Conferences*, Sacramento, California, September 14-17, 1997, Paper No. DETC97/DAC3773.
- [30] Toropov, V. V., Filatov, A. A., and Polynkin, A. A., "Multiparameter Structural Optimization Using FEM and Multipoint Explicit Approximations," *Structural Optimization*, Vol. 6, 1993, pp. 7-14.

- [31] Toropov, V. V., Markin, V. L., and Carlsen, H., "Discrete Structural Optimization Based on Multipoint Explicit Approximations," *Discrete Structural Optimization*, 1994, Germany: Springer-Verlag, pp. 98-107.
- [32] Toropov, V. V., and Carlsen, H., "Optimization of Stirling Engine Performance Based on Multipoint Approximation Technique," *Advances in Design Optimization*, 1994, ASME DE, Vol. 69, No. 2, pp. 531-536.
- [33] Van Keulen, F., Toropov, V. V., and Polynkin, A. A., "Optimization of Geometrically Nonlinear Shell Structures Using Multimeshing and Adaptivity," *Proceedings of the 5th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Panama City Beach, Florida, September 7-9, 1994, Paper No. 94-4361, Part 2, pp. 995-1005.
- [34] Venkataraman, S., and Haftka R. T., "Integration of Finite Element Analyses and Panel Design Program," *Proceedings of the 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Kissimmee, Florida, April 7-10, 1997, Paper No. 97-1052, Part 2, pp. 1215-1224.
- [35] Venter, G., Haftka R. T., and Chirehdast M., "Response Surface Approximations for Fatigue Life Prediction," *Proceedings of the 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Kissimmee, Florida, April 7-10, 1997, Paper No. 97-1331, Part 2, pp. 1383-1396.
- [36] Van Houten, M. H., Schoofs, A. J. G., and Van Campen, D. H., "Response Surface Techniques in Structural Optimization," *First World Congress of Structural and Multidisciplinary Optimization*, Gosler, Germany, May 28-June 2, 1995, pp. 89-94.
- [37] Thacker, B. H., and Wu, Y.-T., "A New Response Surface Approach for Structural Reliability Analysis," *Proceedings of the 33rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference*, Dallas, Texas, April 13-15, 1992, Paper No. 92-2408, Part 2, pp. 586-593.
- [38] Giunta, A. A., Balabanov, V., Haim, D., Grossman, B., Mason, W. H., Watson, L. T., and Haftka, R. T., "Wing Design for a High-Speed Civil Transport Using a Design of Experiments Methodology," *Proceedings of the 6th AIAA/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Bellevue, Washington, September 4-6, 1996, Paper No. 96-4001, Part 1, pp. 168-183.
- [39] Kaufman, M., Balabanov, V., Burgee, S. L., Giunta, A. A., Grossman, B., Mason, W. H., Watson, L. T., and Haftka, R. T., "Variable-Complexity Response Surface Approximations for Wing Structural Weight in HSCT Design," *Journal of Computational Mechanics*, 1996, Vol. 18, No. 2, pp. 112-126.
- [40] Mistree, F., Patel, B., and Vadde, S., "On Modeling Objectives and Multilevel Decisions in Concurrent Design," *Proceedings of the 20th ASME Design Automation Conference*, Minneapolis, Minnesota, September, 1994, pp. 151-161.

- [41] Brown, R. T., and Nachlas, J. A., "Structural Optimization of Laminated Conical Shells," *AIAA Journal*, Vol. 23, No. 5, 1985, pp. 781-787.
- [42] White, K. P., Jr., Hollowell, W. T., Gabler, H. C. III, and Pilkey, W. D., "Simulation Optimization for the Crashworthiness of a Passenger Vehicle in Frontal Collision Using Response Surface Methodology," *SAE Transactions*, 1985, Sec. 3, pp. 798-811.
- [43] Giunta, A. A., Balabanov, V., Burgee, S., Grossman, B., Haftka, R. T., Mason, W. H., and Watson, L. T., "Variable-Complexity Multi-disciplinary Design Optimization Using Parallel Computers," *Computational Mechanics '95—Theory and Application*, Springer, Berlin, 1995, pp. 489-494.
- [44] Mason, B. H., Haftka, R. T., and Johnson, E. R., "Analysis and Design of Composite Curved Channel Frames," *Proceedings of the 5th AIAA/NASA/USAF/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Panama City, Florida, September 7-9, 1994, Paper No. 94-4364, Part 2, pp. 1023-1040.
- [45] Ragon, S., Gürdal, Z., Haftka R. T., and Tzong, T. J., "Global/Local Structural Wing Design Using Response Surface Techniques," *Proceedings of the 38th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Kissimmee, Florida, April 7-10, 1997, Paper No. 97-1051, Part 2, pp. 1204-1214.
- [46] Burgee, S., Giunta, A. A., Balabanov, V., Grossman, B., Mason, W. H., Narducci, R., Haftka, R. T., and Watson, L. T., "A Coarse Grained Parallel Variable-Complexity Multidisciplinary Optimization Paradigm," *International Journal of Supercomputing Applications and High Performance Computing*, Vol. 10, No. 4, 1996, pp. 269-299.
- [47] Roux, W. J., Stander, N., and Haftka, R. T., "Response Surface Approximations for Structural Optimization," *Proceedings of the 6th AIAA/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Bellevue, Washington, September 4-6, 1996, Paper No. 96-4042, Part 1, pp. 565-578.
- [48] Barenblatt, G. I., *Scaling, Self-similarity, and Intermediate Asymptotics*. Cambridge: University Press, 1996.
- [49] Buckingham, E., "Model Experiments and the Form of Empirical Equations," *Transactions of the American Society of Mechanical Engineers*, 1915, Vol. 37, pp. 263-296.
- [50] Peterson, R. E., *Stress Concentration Design Factors; Charts and Relations Useful in Making Strength Calculations for Machine Parts and Structural Elements*. New York: John Wiley & Sons, Inc., 1953.
- [51] Ben-Haim, Y., and Elishakoff, E., *Convex Models of Uncertainty in Applied Mechanics*. Amsterdam: Elsevier Science Publishers, 1990.

- [52] Maglaras, G., Nikolaidis, E., Haftka, R. T., and Cudney, H. H., "Analytical-experimental Comparison of Probabilistic Methods and Fuzzy Set Based Methods for Designing under Uncertainty," *Structural Optimization*, Vol. 13, No. 2, 1997, pp. 69-80.
- [53] Zadeh, L. A., "Fuzzy Sets," *Information and Control*. Vol. 8, 1965, New York: Academic Press, pp. 29-44.
- [54] Liu, T. S., and Huang, G. R., "Fatigue Reliability of Structures Based on Probability and Possibility Measures," *Computer and Structures*, Vol. 45, No. 2, 1992, pp. 361-368.
- [55] Jung, C. Y., and Pulmano, V. A., "Improved Fuzzy Linear Programming Model for Structure Designs," *Computers and Structures*, Vol. 58, No. 3, 1996, pp. 471-477.
- [56] Jensen, H. A., and Sepulveda, A. E., "Fuzzy Optimization of Complex Systems Using Approximation Concepts," *Proceedings of the 5th Panamerican Conference of Applied Mechanics*, San Juan, Puerto Rico, January 2-4, 1997, Vol. 5, pp. 345-348.
- [57] Rao, S. S., "Optimization Using Fuzzy Set Theory," *Structural Optimization: Status and Promise*. Progress in Astronautics and Aeronautics, Vol. 150, 1993, USA: AIAA, pp. 637-661.
- [58] Wu, B., and Young, G., "Modeling Descriptive Assertions Using Fuzzy Functions in Design Optimization," *Proceedings of the 6th AIAA/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Bellevue, Washington, September 4-6, 1996, Paper No. 96-4183, Part 2, pp. 1752-1762.
- [59] Shih, C. J., and Chang, C. J., "Pareto Optimization of Alternative Global Criterion Method for Fuzzy Structural Design," *Computer and Structures*, Vol. 54, No. 3, 1995, pp. 455-460.
- [60] Tornø, T. Y., and Thacker, B. H., "An Efficient Probabilistic Scheme for Constructing Structural Reliability Confidence Bounds," *Proceedings of the 34th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, La Jolla, California, April 19-22, 1993, Paper No. 93-1627, Part 5, pp. 2827-2835.
- [61] Sepulveda, A. E., and Jensen, H., "Comparison of Local and Global Approximation for Reliability-Based Structural Optimization," *Proceedings of the 36th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, New Orleans, Louisiana, April 10-13, 1995, Paper No. 95-1435, Part 4, pp. 2427-2437.
- [62] Wang, L., and Grandhi, R. V., "Intervening Variables and Constraint Approximations in Safety Index and Failure Probability Calculations," *Proceedings of the 5th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and*

- Optimization, Panama City Beach, Florida, September 7-9, 1996, Paper No. 94-4278, Part 1, pp. 253-260.
- [63] Grandhi, R. V., and Wang, L., "Higher-Order Failure Probability Calculation using Nonlinear Approximations," Proceedings of the 37th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Salt Lake City, Utah, April 15-17, 1996, Paper No. 96-1461, pp. 1292-1306.
 - [64] Fox, E. P., "Issues in Utilizing Response Surface Methodologies for Accurate Probabilistic Design," Proceedings of the 37th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Salt Lake City, Utah, April 15-17, 1996, Paper No. 96-1496, Part 3, pp. 1615-1622.
 - [65] Sepulveda, A. E., and Jensen, H., "Fuzzy Optimization Using Approximation Concepts," Proceedings of the 6th AIAA/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization, Bellevue, Washington, September 4-6, 1996, Paper No. 96-4182, Part 2, pp. 1743-1751.
 - [66] Dong, W., and Shah, H. C., "Vertex Method for Computing Functions of Fuzzy Variables," *Fuzzy Sets and Systems*, Vol. 24, 1987, pp. 65-78.
 - [67] Anon., JMP Statistics and Graphics Guide Version 3.1, USA: SAS Institute Inc., 1995.
 - [68] Ott, R. L., *An Introduction to Statistical Methods and Data Analysis*. USA: Wadsworth Inc., 1993.
 - [69] Anon., Microsoft Excel User Manual Version 8.0, USA: Microsoft, 1997.
 - [70] Anon., VMA Engineering, DOT Users Manual Version 4.20, 1995.
 - [71] Anon., Microsoft FORTRAN Powerstation Programmers Guide Version 4.0, USA: Microsoft, 1995.
 - [72] Klir, G., and Yuan, B., *Fuzzy Sets and Fuzzy Logic: Theory and Applications*. USA: Prentice-Hall Inc., 1995.
 - [73] Dubois, D., and Prade, H., *Possibility Theory: An Approach to Computerized Processing of Uncertainty*. New York: Plenum Press, 1988.
 - [74] Davis, S. R., *Learn JAVA Now*. USA: Microsoft Press, 1996.
 - [75] Anon., MSC/NASTRAN Quick Reference Guide Version 68, USA: The Macneal-Schwendler Corporation, 1994.
 - [76] Williams, M. L., "Stress Singularities Resulting from Various Boundary Conditions in Angular Corners of Plates in Extension," *Journal of Applied Mechanics*, Vol. 19, 1952, pp. 526-528.

- [77] Cook, R. D., Malkus, D. S., and Plesha, M. E., *Concepts and Applications of Finite Element Analysis*. New York: John Wiley & Sons, Inc., 1989.
- [78] Timoshenko, S., *Theory of Elastic Stability*. New York: McGraw-Hill Book Company, Inc., 1936.
- [79] Kemp, B. L. and Johnson, E. R., "Response and Failure Analysis of a Graphite-Epoxy Laminate Containing Terminating Internal Plies," Proceedings of the 26th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Orlando, Florida, April 15-17, 1985, Paper No. 85-0608, Part 1, pp. 13-24.
- [80] Fish, J. C., and Lee, S. W., "Tensile Strength of Tapered Composite Structures," Proceedings of the 29th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Williamsburg, Virginia, April, 1988, Part 1, pp. 324-333.
- [81] Dinardo, M. T., and Lagace, P. A., "Buckling and Postbuckling of Laminated Composite Plates with Ply Dropoffs," AIAA Journal, Vol. 27, No. 10, 1989, pp. 1392-1398.
- [82] Curry, J. M., Johnson, E. R., and Starnes, J. H., Jr., "Effect of Dropped Plies on the Strength of Graphite-Epoxy Laminates," AIAA Journal, Vol. 30, No. 2, 1992, pp. 449-456.
- [83] Llanos, A. S., and Vizzini, A. J., "The Effect of Film Adhesive on the Delamination Strength of Tapered Composites," Journal of Composite Materials, Vol. 26, No. 13, 1992, pp. 1968-1983.
- [84] Fish, J. C., and Vizzini, A. J., "Tailoring Concepts for Improved Structural Performance of Rotorcraft Flexbeams," Composites Engineering, Vol. 2, Nos. 5-7, 1992, pp. 303-312.
- [85] Fish, J. C., and Vizzini, A. J., "Delamination of Ply-Drop Configurations," Composite Materials: Testing and Design, Vol. 11, 1993, pp. 323-333.
- [86] Botting, A. D., Vizzini, A. J., and Lee, S. W., "Effect of Ply-Drop Configuration on Delamination Strength of Tapered Composite Structures," AIAA Journal, Vol. 34, No. 8, August, 1996, pp. 1650-1656.
- [87] Dávila, C. G., and Johnson, E. R., "Analysis of Delamination Initiation in Postbuckled Dropped-Ply Laminates," AIAA Journal, Vol. 31, No. 4, 1993, pp. 721-727.
- [88] Harrison, P. N., and Johnson, E. R., "A Mixed Formulation for Interlaminar Stresses in Dropped-Ply Laminates," Proceedings of the 34th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, La Jolla, California, April, 1993, Paper No. 93-1508, Part 3, pp. 1740-1752.

- [89] Rose, C. A., and Starnes, J. H., Jr., "Approximate Analysis for Interlaminar Stresses in Composite Structures with Thickness Discontinuities," Proceedings of the 37th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Salt Lake City, Utah, April 15-17, 1996, Paper No. 96-1497, Part 3, pp. 1623-1638.
- [90] Vizzini, A. J., "Influence of Realistic Ply-Drop Geometries on Interlaminar Stresses in Tapered Laminates," Composite Materials: Fatigue and Fracture, Vol. 5, 1995, pp. 467-485
- [91] Anon., CSAR/NATRAN Quick Look User's Guide Version 97, California: Computerized Structural Analysis & Research Corporation, 1997.
- [92] Sun, C. T., and Li, S., "Three-Dimensional Effective Elastic Constants for Thick Laminates," Journal of Composite Materials, Vol. 22, July, 1988, pp. 629-639.
- [93] Whitney, J. M., and Nuismer, R. J., "Stress Fracture Criteria for Laminated Composites Containing Stress Concentrations," Journal of Composite Materials, Vol. 8, 1974, pp. 253-265.
- [94] Gibson, R. F., *Principles of Composite Material Mechanics*. USA: McGraw-Hill, Inc., 1994.
- [95] Brewer, J. C., and Lagace, P. A., "Quadratic Stress Criterion for Initiation of Delamination," Journal of Composite Materials, Vol. 22, 1988, pp. 1141-1155.

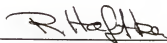
BIOGRAPHICAL SKETCH

Gerhard Venter was born in Bothaville, South Africa on September 1, 1970. He received a Bachelor of Science degree in Mechanical Engineering from the University of Pretoria in December, 1992 and the Honors degree¹ in Mechanical Engineering from the same university in December, 1993. He also received the Rentmeester award for the best Honors student from the faculty of engineering at the University of Pretoria.

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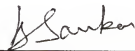
¹ Honors degree is a graduate degree consisting of eight graduate courses

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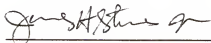
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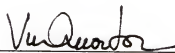
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